REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
8 Apr 96

3. REPORT TYPE AND DATES COVERED

4. TITLE AND SUBTITLE
The Effect of Distributed Practice Homework on Precalculus Achievement at a Military Academy

5. FUNDING NUMBERS

6. AUTHOR(S)

Marie Agnes Revak

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
AFIT Student Attending:
Florida Institute of Technology

8. PERFORMING ORGANIZATION REPORT NUMBER
96-015D

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
DEPARTMENT OF THE AIR FORCE
AFIT/CI
2950 P STREET, BLDG 125
WRIGHT-PATTERSON AFB OH 45433-7765

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release IAW AFR 190-1
Distribution Unlimited
BRIAN D. GAUTHIER, MSgt, USAF
Chief Administration

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

19960530 105

14. SUBJECT TERMS

15. NUMBER OF PAGES
170

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT

18. SECURITY CLASSIFICATION OF THIS PAGE

19. SECURITY CLASSIFICATION OF ABSTRACT

20. LIMITATION OF ABSTRACT

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★U.S.GPO:1990-0-273-271

The Effect of
Distributed Practice Homework
on Precalculus Achievement
at a Military Academy

by

Marie Agnes Revak

Bachelor of Arts
in Mathematics
Rutgers University
1978

Master of Arts
in Mathematics Education
Glassboro State College
1985

Specialist in Education
in Mathematics Education
Florida Institute of Technology
1995

A dissertation
submitted to the Graduate School of
Florida Institute of Technology
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy
in
Mathematics Education

Melbourne, Florida
April 8, 1996

The undersigned committee
having examined the attached dissertation

"The Effect of
Distributed Practice Homework
on Precalculus Achievement
at a Military Academy"
by
Marie Agnes Revak

hereby indicates its unanimous approval

K. Kelly-Benjamin, Ph.D., Major Advisor
Associate Professor, Science Education

P. B. Horton, Ph.D.
Professor, Science Education

R. E. Enstice, Ph.D.
Assistant Professor, Science Education

J. Yellen, Ph.D.
Associate Professor, Applied Mathematics

Paula S. Krist, Ph.D.
Assistant Professor, Science Education

R. H. Fronk, Ph.D.
Professor, Science Education

ABSTRACT

The Effect of Distributed Practice Homework on

Precalculus Achievement at a Military Academy

by: Marie Agnes Revak

MAJOR ADVISOR: Kathy Kelly-Benjamin, Ph.D.

This study investigated the main effect of distributive practice homework on achievement in Precalculus. This study also investigated the aptitude-treatment interaction effects of distributed practice homework $\times$ mathematics achievement and distributed practice homework $\times$ mathematics anxiety on achievement in Precalculus.

The sample consisted of 351 United States Air Force Academy cadets (experimental $\underline{n}$ = 161, control $\underline{n}$ = 190), all in their first semester of their first year of college. Three instructors taught experimental group sections and five instructors taught control group sections. Instructor experience level was equalized across the groups.

An algebra/trigonometry placement exam was used as the measure of prior mathematics achievement. A subset of the Math

Anxiety Rating Scale developed by Alexander and Martray (1989) was used as the measure of mathematics anxiety. Precalculus grades (4 hourly exams, a final exam, and the final course percentage grade) were used to measure achievement.

Data were analyzed using hierarchical multiple regression aptitude-treatment interaction techniques.

Distributed practice homework significantly bolstered the achievement of students on 4 of the 6 measures of Precalculus achievement (3 hourly exams and the final course percentage grade) without regard for prior math achievement or math anxiety (alpha = .05). Other findings indicate that as study time increased, the achievement difference between the experimental and control groups decreased. An analysis of homework error trends indicated that the treatment group made fewer repeat errors than the control group.

Different variations of the distributed practice model should be investigated across a wide variety of students, institutions, and mathematics courses. The contribution of other variables, such as motivation, attitude, and study habits should also be examined.

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KEYWORDS

aptitude-treatment interaction

college mathematics

continuous review

distributed practice

homework

lag effect

mathematics anxiety

precalculus

prior achievement

spaced review

spacing effect

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ACKNOWLEDGMENT

Although a dissertation is presumed to be proof of a person's ability to conduct independent research, this work required the assistance and support of many individuals.

My husband, Ray Yelle, made career sacrifices and forfeited a normal family life for two years so that I might achieve my goal. My parents, Bill and Marie Revak, passed along their value for education and have supported my educational and career efforts from Kindergarten to Ph.D. and hoagie maker to Assistant Professor. My in-laws, Donald and Vivian Yelle, helped tremendously by caring for the kids during the most stressful times of my program.

My advisor, Dr. Kathy Kelly-Benjamin (K-B) has supported this research effort from its earliest conception. With her guidance, I have grown as a student, teacher, and researcher. I look forward to her continued mentoring as I cross the threshold from student to colleague.

I carefully selected my committee members so that I might be challenged, guided, and supported. Be it course work, comprehensive exams, or research, Dr. Phil Horton has never failed to challenge me. Because of his high expectations, I know I have a dissertation I can be proud of. Dr. Jay Yellen has challenged me mathematically and has set an example as a mathematician who values education. Dr. Dick Enstice helped our cadre become a cohesive unit and provided the guidance and support necessary for us to meet the challenge of the comprehensive exams. I am pleased to have finished my program with all of my original committee members, plus one. Through her teaching, research, and advising, Dr. Paula Krist has set an example of what a "new" Doctor should be.

My fellow Doctoral students have become my good friends. Hettie Buck has been my study partner and sounding board. She has more recently become my local presence on campus while I conducted my research in Colorado. Ali Eryilmaz has served as a study partner and statistical consultant. Tom Dreschel and Mike Witiw were important study group members and were always there to support and encourage me. I am looking forward to long friendships and collaborative research projects.

The United States Air Force provided the funding for my academic program. Colonel Steve Gordon provided the opportunity. Without the support of past and present members of the math department at the Air Force Academy, this work would not have been possible. In support of this research, Colonel Dan Litwhiler offered guidance and support along with a big batch of faculty and students. Cindy Brown and Paul Simonich designed a top-notch student-centered course with high quality exams, projects, and activities. Although new to teaching, Glen Wiggy, Scott Frickenstein, and Harmon Lewis proved themselves to be capable and enthusiastic teachers. Father Bob Dunn, Jim Przybysz, and Colonel Dan Litwhiler supported the new instructors by sharing their knowledge and expertise. Also important was the advise and encouragement of Lt. Colonel Tuiren Bratina, Lt. Colonel Carl Crockett, Lt. Colonel Bill Kiele, Dr. Jay Sherman, Dave Cribb, Lynn Young, Trish Stetler, Marilyn Sacchetta, and all of the other members of DFMS. I am looking forward to again being part of their prestigious team.

This work is dedicated to my daughters,

Brianna Nicole Yelle
and
Cassandra Marie Yelle

with special thanks for their pleasant diversions
and unconditional love

CHAPTER 1

INTRODUCTION

According to the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 1989), "today's society expects schools to insure that all students have an opportunity to become mathematically literate" (p. 5). In the past, courses such as algebra, precalculus, and calculus have acted as gatekeepers, allowing only the best and the brightest to pass through to higher levels of science and mathematics (National Commission on Excellence in Education, 1983; NCTM, 1989; National Research Council, 1989; Sells, 1978). While these gatekeepers have been keeping students out of the mathematics pipeline, there has been an increase in the number of university degree programs requiring Calculus (Waits & Demana, 1988).

Success in mathematics is often cyclic in nature, with new successes influenced by prior achievement (Cooper & Robinson, 1989; Covington & Omelich, 1987). Experience with a method or medium of instruction often helps students learn new content where the same method or medium is used (Becker, 1970; Koran & Koran, 1984). Although the influence of ability is strong, non cognitive variables also have a role in predicting student achievement. Cronbach and Snow (1977) recommended that researchers study affective variables in addition to scores on conventional achievement tests because a student's response to instruction is conditioned by all of his or her characteristics. According to McLeod (1992), attention to the affective domain is imperative if the researcher sees the learner as someone who is actively involved in the construction of knowledge.

Although it is not known whether mathematics anxiety causes poor mathematics achievement or whether the reverse is true (Birenbaum & Gutvirtz, 1993; Cooper & Robinson, 1989; Covington & Omelich, 1987; Fulkerson, Galassi, & Galassi, 1984; Hembree, 1990; McKeachie, 1988; Meece, Wigfield, & Eccles, 1990; Naveh-Benjamin, McKeachie, & Lin, 1987; Richardson & Woolfolk, 1980; Tobias, 1990), it is clear that math anxiety correlates negatively with achievement (Adams & Holcomb, 1986; Berenson, Carter, & Norwood, 1992; Blum-Anderson, 1992; Coleman, 1991; Cooper & Robinson, 1989; Covington & Omelich, 1987; Frary & Ling, 1983; Giangrasso, 1981; Gliner, 1987; Hembree, 1990; Koran & Koran, 1984; Lawson, 1993; Richardson & Suinn, 1972; Schneider & Nevid, 1993; Simon, 1992).

The National Research Council declared that "mathematics must become a pump rather than a filter in the pipeline of American education" (1989, p. 7). The NCTM Curriculum and Evaluation Standards (1989) have charged the community of educators to explore new options to ensure success in mathematics for all students. A review of the literature indicates that efforts toward ensuring mathematical power for all students will require an examination of the impact of prior math success and affective variables, such as anxiety, on mathematics achievement.

Research on the effect of mathematics homework on achievement has produced mixed results (Austin, 1979; Featherstone, 1985; Suydam, 1985). Although the National Commission on Excellence in Education (1983) warned that time spent on homework is often used ineffectively, the caliber and composition of homework has only rarely been studied (Strother, 1984). Two studies have investigated different models for assigning college mathematics homework (Hirsch, Kapoor, & Laing, 1982, 1983; Peterson, 1970) with mixed results. These studies were based on the aspect of information processing learning theory known as the spacing effect.

The spacing effect is the phenomenon in which "material encountered on two different occasions with a lapse of time between the encounters is remembered better than material studied [for] an equal amount of time on one occasion" (Krug, Davis, & Glover, 1990, p. 366). The spacing effect has a long history in cognitive psychology and education research and is known by many names including lag effect, distributed practice, continuous review, and spaced review (Cuddy & Jacoby, 1982; Dempster, 1988; Krug et al., 1990; Melton, 1970; Reynolds & Glaser, 1964; Toppino & Gracen, 1985; Underwood, 1961). According to Dempster (1988), although distributed practice is "one of the most remarkable phenomena to emerge from laboratory research" (p. 627), there is little evidence that its potential has been realized in applied settings.

Statement of the Problem

Homework is commonplace in college mathematics courses, yet, with the exception of research investigating Saxon's incremental continuous review method (Abrams, 1989; Denson, 1989; Gianniotes, 1989; Johnson & Smith, 1987; Klingele & Reed, 1984; Parker, 1990; Reed, 1983; Rentschler, 1995; Roberts, 1994; Saxon, 1982b), little research has been conducted on the content or quality of mathematics homework. Additionally, there is a lack of research investigating the differential effectiveness of homework for students with varying aptitudes (Austin, 1979; Featherstone, 1985; Hirsch et al., 1982, 1983; Koehler & Grouws, 1992; Peterson, 1971; Suydam, 1985). By assigning homework problems related only to the most current course topics, mathematics educators have ignored the findings of cognitive psychology research recommending spaced over massed practice (Dempster, 1988; Reynolds & Glaser, 1964).

Research shows that students with low prior mathematics achievement, high mathematics anxiety, or both have a low probability of success in college mathematics (Berenson et al., 1992; Blum-Anderson, 1992; Cooper & Robinson, 1989; Frary & Ling, 1983; Lawson, 1993; Richardson & Suinn, 1972; Waits & Demana, 1988). The NCTM and National Commission on Excellence in Education have put forth a challenge to develop new strategies for reaching at-risk students. Studies exploring these options should consider both cognitive and affective variables (Corno & Snow, 1986; Cronbach & Snow, 1977; Eastman & Dietz, 1978; Gehlbach, 1979; McLeod, 1990; Snow, 1989, 1992; Tobias, 1976, 1989).

Reviews of aptitude-treatment interaction (ATI) research suggest that low ability students may benefit from high levels of instructional support (Tobias, 1976, 1989) and highly anxious students may benefit from highly organized instructional materials (Koran & Koran, 1984; Tobias, 1989; Sieber, O'Neill, & Tobias, 1977). This dissertation explores distributed practice homework assignments as one way to provide the instructional support and task organization necessary to increase the mathematics achievement of students with low prior mathematics achievement, high levels of mathematics anxiety, or both.

Research Hypotheses

(1) The type of homework assignment will not significantly affect student achievement in Precalculus. That is, there will not be a significant main (or average) effect due to the distributed practice treatment.

(2) The type of homework assignment will have a significantly different effect for students with differing levels of prior mathematics achievement. That is, the regression lines representing the treatment and control groups will have significantly different slopes. Distributed practice homework assignments will benefit students with low prior mathematics achievement significantly more than they benefit students with high prior mathematics achievement.

(3) The type of homework assignment will have a significantly different effect for students with differing levels of mathematics anxiety. That is, the regression lines representing the treatment and control groups will have significantly different slopes. Distributed

practice homework assignments will benefit students with high mathematics anxiety significantly more than they benefit students with low mathematics anxiety.

(4) The type of homework assignment will not have a significantly different effect for students with differing combinations of prior achievement and mathematics anxiety. That is, the three way-interaction of prior achievement, anxiety, and treatment will not explain a significant proportion of variance in achievement above what has been explained by prior achievement, anxiety, the treatment, and the two-way interactions (Prior Achievement × Treatment and Anxiety × Treatment).

Null Hypotheses

(1) Distributed practice homework assignments will not significantly affect student achievement in precalculus.

(2) The interaction of prior mathematics achievement and the distributed practice treatment will not explain a significant proportion of variance in Precalculus achievement above what has been explained by prior mathematics achievement, mathematics anxiety, and the treatment.

(3) The interaction of mathematics anxiety and the distributed practice treatment will not explain a significant proportion of variance

in Precalculus achievement above what has been explained by prior mathematics achievement, mathematics anxiety, and the treatment.

(4) The three-way interaction of prior mathematics achievement, mathematics anxiety and the distributed practice treatment will not explain a significant proportion of variance in Precalculus achievement above what has been explained by prior achievement, anxiety, the treatment, and the two-way interactions (Prior Achievement × Treatment and Anxiety × Treatment).

Definitions

Achievement. The accumulation of Precalculus skills and concepts as measured by four hourly examinations, a final exam, and the final course percentage grade. All exams were cumulative and were designed by the faculty of the Department of Mathematical Sciences at the United States Air Force Academy.

Aptitude. "Any characteristic of a person that forecasts his [or her] probability of success under a given treatment" (Cronbach & Snow, 1977). In this study, two aptitudes were measured and examined: prior mathematics achievement and mathematics anxiety.

Distributed Practice Homework Assignments. Homework assignments designed so that skills and concepts taught in previous lessons are practiced throughout the semester. The method of distributed practice is also referred to as spaced review or continuous review. The distributed homework model described by Peterson (1971) and Hirsch et al. (1982) was employed.

Mathematics Anxiety. Feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematics problems in real world and academic situations (Richardson & Suinn, 1972). Mathematics anxiety was measured by an abbreviated version of the Mathematics Anxiety Rating Scale (MARS), college and adult version (Alexander & Martray, 1989).

Prior Mathematics Achievement. Mathematics achievement as measured by raw scores on the Algebra/Trigonometry Placement exam authored by the faculty of the Department of Mathematical Sciences at the United States Air Force Academy. The multiple choice exam consisted of 25 algebra items and 10 trigonometry items.

Treatment. "Any manipulable variable" (Cronbach & Snow, 1977). In this study, the treatment consisted of distributed practice homework assignments.

CHAPTER 2

REVIEW OF RELATED LITERATURE

This literature review situates the proposed study into the existing research in several areas. The following bodies of available research have been reviewed and summarized: homework, distributed practice, aptitude-treatment interaction (ATI), mathematics anxiety, mathematics prior achievement, and relevant studies conducted at military academies.

Homework

Most past research on home study has been of the homework versus no homework variety. For the most part, researchers failed to describe the homework that had been assigned (Strother, 1984) and also failed to separate the effects of homework from the effects of other variables such as study skills and student characteristics (Keith, 1982). Austin's (1979) meta-analysis of eight post-1960 studies of mathematics homework drew the following conclusions: (a) for grades four through ten, homework seems preferable to no homework; (b) homework may produce a cumulative effect; (c) the value of routine drill homework is limited; (d) there does not appear to be a relationship between homework and attitude; (e) comments on homework may improve student achievement but every homework problem need not be graded; (f) homework appears to have a positive impact on computational skills; and (g) the effects of homework on problem solving are unclear. Paschal, Weinstein, and Walberg (1984) reached a similar conclusion in their synthesis of homework research. They reported that 85% of the comparisons favored the homework groups (weighted grand mean effect size = .359). They also found that larger effects on achievement were found when homework was graded or commented on. Cooper's (1989) synthesis of homework research concluded that "there was strong evidence that it is better to distribute material across several assignments rather than have homework concentrate only on material covered in class that day" (p. 89).

Strother (1984) provided a thorough accounting of the history of homework research, noting that the topic of homework often inspired controversy. Indeed, homework appears to fall in and out of favor with the public. Cooper (1989) reported that homework gained approval in the 1950s after the launch of Sputnik. Homework lost its favor in the 1960s and 1970s only to regain favor in the 1980s after the publication of the report A Nation at Risk by the National Commission on Excellence in Education (1983).

Although recent experimental research on homework is scant, opinions about mathematics homework abound. Practitioners commonly recommend mathematics homework that promotes problem solving, involves a wide range of activities, provides an opportunity for further exploration, and is tailored to individual student needs (Burns, 1986; Featherstone, 1985; McLean, 1986; Suydam, 1985). Mathematics educators often advocate homework problems requiring analysis, evaluation, synthesis, and the transfer of knowledge to new situations. They also recommend the inclusion of new and different kinds of problems (Braswell, 1985). In addition, education experts advise teachers to stress the importance of homework (Braswell, 1985; McLean, 1986; Orsetti, 1984) and to assign homework that can be worked in a reasonable amount of time (Burns, 1986; McLean, 1986). Teachers are encouraged to plan homework as carefully as they plan classroom instruction (Strother, 1984) and choose homework that reflects curricular goals (Suydam, 1985).

The few homework studies involving high school and college students have produced mixed results. In his investigation using a remedial algebra class, Ryder (1982) concluded that there was no significant difference in achievement between the homework and no homework groups. The results of Suciu's (1991) study contradicted Ryder's results. In all five introductory college mathematics courses studied, Suciu found that "completion of assignments and amount of practice correlated positively with success" (p. 748-A). Keith (1982) analyzed the data from the National Center for Education Statistics' High School and Beyond longitudinal study (n = 20,364 high school seniors) and concluded that an increase in time spent on homework had a positive correlation with high school grades. Keith also reported that low ability students could achieve average grades by doing one to three hours of homework each week. Likewise, Senn (1984) found that general chemistry students who completed assigned homework achieved significantly higher scores on the posttest than students who were not assigned homework problems. Although Ryder found no empirical evidence to support homework, student surveys indicated that they believe "assigned homework is necessary" (p. 711-A).

Distributed Practice

Research on distributed practice is situated in information processing learning theory (Ausubel, 1966). For over 25 years, cognitive psychology research has documented the benefit of spaced practice (Cuddy & Jacoby, 1982; Krug et al., 1990; Melton, 1970;

Modigliani, 1976; Rea & Modigliani, 1985; Toppino & Gracen, 1985; Underwood, 1961). The most typical finding of this research was that as spacing increased, retention also increased. However, most research pertaining to the spacing effect has investigated the learning of simple word or number lists with time lags measured in seconds. Although the spacing effect is "one of the most robust phenomena discovered in memory research" (Rea & Modigliani, 1985, p. 11), results from cognitive psychology experiments do not necessarily transfer to complex learning tasks with longer spacings between reviews (Reynolds & Glaser, 1964). According to Dempster (1988), studies conducted from a basic research perspective and those conducted from an applied perspective frame two distinct research strands.

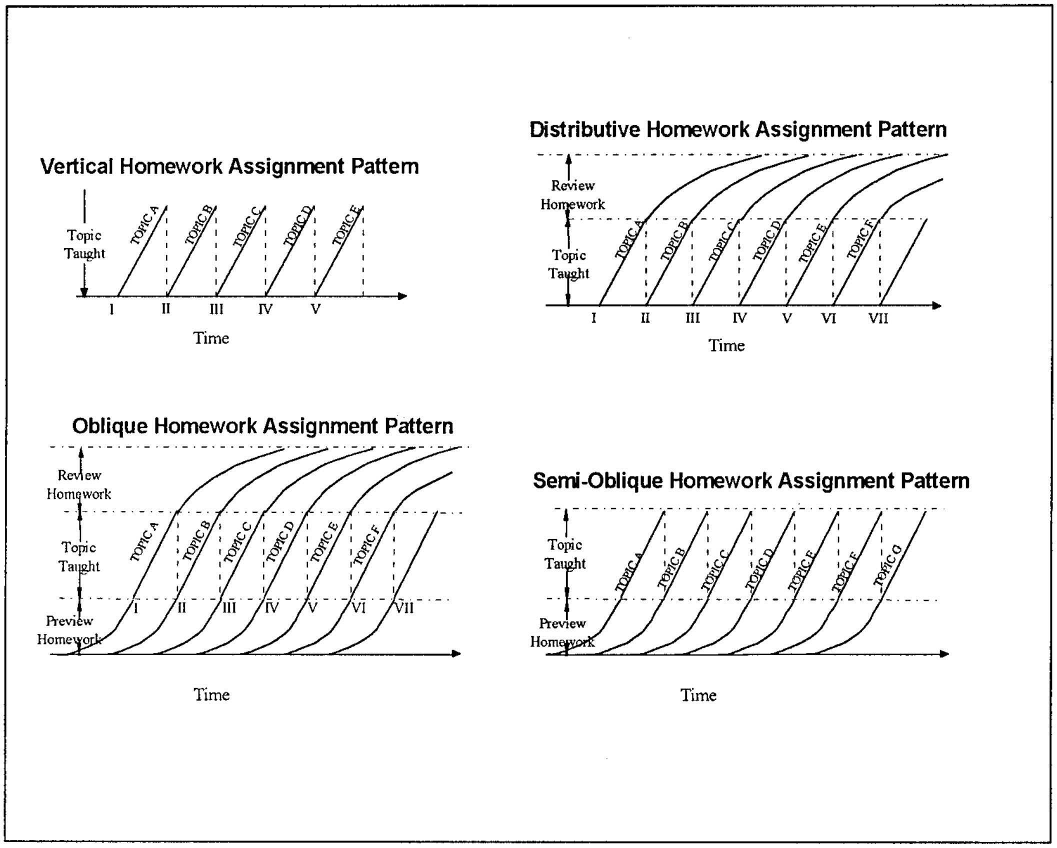
In an attempt to move distributed practice research from the laboratory to the classroom, Reynolds and Glaser (1964) investigated the effect of spaced review upon retention of meaningful learning of a complex task. In their study using an eighth grade science unit, Reynolds and Glaser found that spaced review resulted in higher short term and long term retention rates regardless of the length of the retention interval. Grote (1992) reported similar achievement gains when he applied distributed practice to high school physics homework assignments.

Peterson (1971) drew heavily from information processing theory when he examined models for incorporating homework into the teaching of mathematics. In this work, Peterson defined four basic homework assignment patterns: vertical, oblique, distributive, and semi-oblique. Traditional homework assignments follow the vertical assignment pattern; homework problems are related to the most recent course topic(s). Under the oblique pattern, homework assignments consist of exploratory, practice, and review problems. The exploratory problems prepare students for future topics, current topic(s) offer opportunities for practice, and review reinforces previous topic(s). Homework assignments under the distributive pattern include practice and review problems. Finally, the semi-oblique model combines exploratory and practice problems. Figure 1 captures the essence of Peterson's homework models.

Research comparing the effects of distributed practice homework and traditional homework has produced mixed results. Peterson (1970) studied the effect of supplementary exploratory exercises ($\underline{n}$ = six eighth grade classes) and found that students who attempted at least 50% of the assigned exploratory exercises had

Figure 1.

Vertical, oblique, distributive, and semi-oblique homework assignment patterns.



Note. From "Four Organizational Patterns for Assigning Mathematics Homework," by J. C. Peterson, 1971, School Science and Mathematics, 71, p. 592. Reprinted with permission (see Appendix D).

significantly higher achievement and retention scores than a group that was not assigned supplementary homework. Similarly, Friesen's (1975) experiment (N = 143) employed a homework model that combined exploratory, current, and review exercises and found significant differences in overall achievement and retention favoring the experimental group. Butcher (1975) conducted two independent studies comparing the effects of distributed and massed algebra assignments. In the first study, Butcher found a significant difference in achievement favoring the distributed practice group on one out of two measures of achievement; no difference between the groups was found in the second study. When retention was measured, Butcher found no significant difference between the groups in either study. Similarly, studies by Camp (1973), Dadas (1976), and Weaver (1976) reported no significant differences in achievement when the experimental treatment consisted of distributed practice homework. Due to the short duration of the studies, the results reported by Butcher, Camp, Dadas, and Weaver should be taken cautiously.

The incremental continuous review method of assigning mathematics homework advocated by Saxon (1982b, 1984) follows Peterson's (1971) distributed homework pattern. The publication of Saxon's (1982a) Algebra I textbook provoked a number of studies comparing Saxon's incremental continuous review method of assigning homework to more traditional methods. Under Saxon's (1984) model, homework consisted of four problems based on the newest material with other problems based on earlier lessons.

The majority of mathematics homework research has focused on the academic achievement of students using the Saxon series of textbooks. This research has also produced mixed results. For example, Saxon (1982b) reported favorable results for a group of ninth graders (n = 541) using a prototype of his yet to be published Algebra I textbook. Reed's study using remedial college algebra students found that the Saxon continuous review group (n = 296) outscored the control group (n = 299) on the departmental final exam and the Basic Algebra Test published by the Mathematical Association of America (Reed, 1983; Klingele & Reed, 1984). Similarly, Parker (1990) reported small but significant gains by the seventh-grade Saxon group on two out of three portions of the Stanford Achievement Test.

Roberts' (1994) study compared achievement test gain scores over two years to determine the impact of a Saxon mathematics program (N = 185). Although Roberts reported higher gains for the experimental group, the difference was not statistically significant. Rentschler (1995) reported a significant difference in computation skills favoring a sixth grade class using the Saxon textbook but no differences between the Saxon group and the control group in concept or application scores. A study involving six Algebra I classes found no significant differences in the achievement of the Saxon group when compared to the control group (Gianniotes, 1989). Gianniotes offered the possible explanation that the Saxon group completed fewer assignments than the control (massed practice) group. In a study with similar results, Denson (1989) concluded that "neither massed traditional nor distributed practice was found to be more effective than the other" (p. 3173-A). Denson performed a total of 60 analyses of covariance and found that the control group (n = 180) outscored the distributed practice group (n = 180) in two subcategories, but attributed the difference to topic coverage. Denson's results should be taken cautiously due to the high experimentwise error rate caused by the large number of statistical tests conducted.

A study by Abrams (1989) involving 300 students found the achievement of the traditional group to be superior to that of the Saxon group. The traditional group (massed practice) outscored the experimental group (continuous review) on three out of four measures of achievement. Abrams found that the experimental treatment was weakened due to the teachers' adaptation of Saxon's content and method to meet their own instructional styles. Finally, in a study utilizing superior Algebra I students, Johnson and Smith (1987) reported that the control group (n = 136) significantly outscored the Saxon group (n = 140) on one of three achievement scores.

Several related learning theories concerning spaced review have been posited. One such theory is the encoding variability hypothesis (McDaniel & Masson, 1985; Cuddy & Jacoby, 1982) in which multiple encodings result in multiple retrieval paths producing easier accessibility to memory.

In his article, "Synthesis of Research on Reviews and Tests," Dempster (1991) promoted spaced review (distributed practice) over massed review for deepening students' learning. He theorized that spaced repetitions encourage constructive mental processes. His reconstruction or accessibility hypothesis is similar to the encoding variability hypothesis. Dempster's premise is that repetition forces a student to try to retrieve or access information from memory. He theorized that shorter spacings between repetitions allows for easier accessibility of stored information. Dempster also postulated that spaced repetitions result in a deeper understanding of the topic. According to Cuddy and Jacoby (1982), for a repetition to be most effective, it may be necessary for an item to be partially forgotten or not easily accessible.

Ausubel (1966) endorsed spaced review with several conjectures. First, distributed practice is associated with additional opportunity for cognitive interaction with the learning materials. By increasing the number of cognitive interactions, the probability that the learning will be embodied in the students' existing knowledge structure is also increased. Ausubel (1966) also hypothesized that additional study trials allow the learner to test his or her knowledge, clarify ambiguities, correct misconceptions, and identify areas of weakness. Finally, Ausubel (1966) speculated that students are sensitized to new knowledge in the initial contact with the learning material, making the material easier to understand in subsequent encounters.

Peterson's (1971) distributive pattern for assigning homework appears to be based on Thorndike's (1932/1971) theory that early reviews should be longer and occur soon after initial learning with later, shorter reviews occurring at increasingly longer intervals.

Although the benefit of the spacing effect is well documented in memory research (Krug et al., 1990; Melton, 1970; Rea & Modigliani, 1985; Toppino & Gracen, 1985; Underwood, 1961), distributed practice has not always produced significant achievement gains. For example, whereas Reed found that college-level remedial algebra students practicing continuous review had a significantly higher achievement than a control group (1983; Klingele & Reed, 1984), Johnson and Smith's (1987) study employing high school Algebra I students reported the opposite finding. Interestingly, the students participating in the Klingele and Reed study were remedial students and those participating in the Johnson and Smith study were described as "well above the national norm in initial mathematics achievement" (p. 100). This difference in the effect of spaced review homework assignments points to the possibility of an interaction between mathematics aptitude and the distributed practice treatment; that is, a possible aptitude-treatment interaction.

Aptitude-Treatment Interaction

According to Cronbach and Snow, "an interaction is said to be present when a situation has one effect on one kind of person and a different effect on another" (1977, p. 3). Snow and Lohman defined aptitude as "readiness to learn from a particular instructional treatment" (1984, p. 349) and treatment as any manipulable variable.

Salomon (1972) described aptitude-treatment interaction (ATI) research as accomplishing two functions: improving instruction and "developing explanatory principles concerning the nature of instruction" (p. 328). Three models have been proposed for using ATI to improve instruction: (a) remediation, (b) compensation, and (c) capitalization (Koran & Koran, 1984; Salomon, 1972). Remedial programs attempt to overcome learner deficiencies when learning is sequentially ordered (Salomon). Under the compensatory model, treatments interact with aptitudes by circumventing their debilitating effects without trying to improve them (Salomon). When the preferential or capitalization model is employed, instructional treatments are designed to make the most of student capabilities (Salomon). The remedial and compensatory ATI models tend to benefit low aptitude students while weakening the performance of high ability students. High aptitude students tend to benefit most from the preferential model (Salomon).

Some saw the goal of ATI research in education as tailoring instruction to particular student aptitudes (Becker, 1970; Bracht, 1970; Corno, 1988; Glaser, 1972; Hirsch et al., 1982, 1983; Koran & Koran, 1984; Tobias, 1979). In his comprehensive review of ATI research in mathematics education, Holtan (1982) supported the use of ATI studies to address the needs of individual students.

ATI research has uncovered few replicated interactions on which to base wide prescriptive treatments (Becker, 1970; Holtan, 1982;

Radatz, 1979; Salomon, 1972; Tobias, 1976, 1979, 1989). Glaser warned that "few or no ATI effects have been solidly demonstrated; the frequency of studies in which appropriate interactions have been found is low; and the empirical evidence found in favor of such interactions is often not very convincing" (1972, p. 8). Salomon (1972) warned that many ATIs were "simply uninterpretable" (p. 327). Tobias (1976) concluded that ATIs may be so specific as to vary according to content area. Similarly, Snow (1977a) warned that interactions among and between individual difference variables and instructional conditions can be so complex that generalizations are not possible. In his attempt to explain the complexity of interactions, Cronbach stated, "once we attend to interactions, we enter a hall of mirrors that extends to infinity" (1975, p. 119).

Because of this complexity, Snow (1977a) conceded that ATI does not lend itself to general theory. He instead advocated <u>local</u> theory. Cronbach (1975) also advocated giving proper consideration to local conditions and advised researchers to treat generalizations as working hypotheses and not conclusions. "The psychologist can describe the conditions under which his generalizations have held, or the domain of which they provide an actuarial summary. He [or she] cannot often state the boundaries defining how far they will hold" (Cronbach, 1975, p. 125).

Seeking to apply ATI results, Gehlbach (1979) spoke out against Snow's recommendation to develop local theory and envisioned a different goal for ATI research. According to Gehlbach, a principal aim of ATI research should be to find ways to eliminate ATIs because steep aptitude-treatment regressions point to possible inequities in public education. Gehlbach (1979) advocated <u>AT correction</u> - instructional treatments designed and modified in such a way as to reduce the slope of the AT regression without lowering the mean achievement level. Snow's later thoughts about ATI were in harmony with Gehlbach, advocating combined treatments with the aim of eliminating ATIs (Snow, 1992).

Despite the differences of opinion as to the goal of ATI research, educational researchers agree on the importance of studying the impact of affective as well as cognitive variables. According to Snow, "any aspect of a person that can predict his or her response to instruction ought to be examined as relevant to important personal and instructional goals" (1992, p. 9).

Snow (1977a) advocated the use of some measure of general ability in all instructional research and evaluation. Tobias (1976) agreed that prior achievement holds an important place in ATI research. Koran and Koran (1984) urged ATI researchers to consider the nature of the psychological processes required by the instructional treatments when selecting affective variables. In the past, general ability and anxiety variables have produced fairly consistent results in ATI studies (Corno & Snow, 1986; Koran & Koran, 1984).

Whenever affective traits are considered, researchers should expect that the regression of the trait will vary with ability. This statement holds true for the anxiety construct. Cronbach and Snow (1977) assert that the anxiety experienced by an individual depends on the difficulty he or she has with a task. Task difficulty depends on an individual's ability and the characteristics of the task. Therefore, a complex task is more likely to create anxiety in persons of low ability than in more able persons (Cronbach & Snow). For these reasons, Cronbach and Snow recommended the analysis of the three-way interaction of ability, anxiety, and treatment when investigating achievement.

Very few ATI studies have employed distributed practice as an instructional treatment. Hirsch et al. (1982, 1983) studied the interaction of mathematical ability and distributed practice homework assignments on achievement in first semester college calculus. The researchers applied a linear regression model to each of five measures of calculus achievement (four tests and a final exam) spaced throughout the semester. They found significant differences between the slopes of the regression lines representing the treatment ($\underline{n}$ = 24) and control groups ($\underline{n}$ = 28) on three of the five measures of achievement and a non-significant interaction on the remaining two measures. Hirsch and his colleagues concluded that "the pay-off for the distributive model may be in terms of improved performance of the student of average or below-average mathematics ability" (1983, p. 56). In a similar study, Revak (1994) studied the interaction of prior mathematics achievement and distributed practice homework assignments. The subjects in this study were Calculus II college students (treatment $\underline{n}$ = 55, control $\underline{n}$ = 416). She reported non-significant ordinal interactions on three of four measures of achievement. Revak found that distributed practice problem sets favored students with low prior mathematics achievement, but the interaction effect was not statistically significant. In Abrams' (1989) experiment using the Saxon textbook, the interaction between treatment and mathematical ability was not statistically significant.

Mathematics Anxiety

Tobias described anxiety as "one of the major psychological variables in education" (1979, p. 573). The effects of mathematics anxiety are far reaching. Four decades ago, Dreger and Aiken (1957) reported that about one-third of college students enrolled in basic mathematics classes suffered from number anxiety. More recently it has been reported that up to half of all college students are intensely anxious about mathematics at one time or another (Adams & Holcomb, 1986; Betz, 1978; Cope, 1988; Richardson & Suinn, 1972). In addition, many of the students who suffer from mathematics anxiety do not suffer from other apprehensions (Richardson & Suinn). For many, the feeling of utter defeat associated with math anxiety is difficult or impossible to overcome (Tobias, 1978).

Mathematics anxiety often produces a feeling of panic during testing situations (Blum-Anderson, 1992; Cooper & Robinson, 1989; Lawson, 1993). Blum-Anderson cited Mandler (1984) in stating that cognitive processing ability is often thwarted or arrested by the physical reactions associated with panic.

Studies of college students by Coleman (1991), Cope (1988), and D'Ailly and Bergering (1992) found a direct relationship between level of mathematics anxiety and math avoidance. Meece and her colleagues (1990) found that, in their junior high school sample, mathematics anxiety had an indirect, negative effect on subsequent performance in mathematics and on intentions to take more mathematics courses. Research by Betz (1978), Coleman (1991), Frary and Ling (1983), and Resnick, Viehe, and Segal (1982), and a meta-analysis by Hembree (1990) reported negative relationships between mathematics anxiety and mathematics background. High negative correlations were also found between mathematics anxiety and confidence in learning mathematics (Goolsby, Dwinell, Higbee, & Bretscher, 1988). Additionally, college students with higher levels of mathematics anxiety were found to have lower college grade-point-averages (Dreger & Aiken, 1957; Frary & Ling, 1983; Gliner, 1987, Hembree, 1990).

Simon (1992) reported that student emotions affected the continuance and completion of algebraic tasks. In Giangrasso's (1981) study, students with high mathematics anxiety employed a lesser number and a more limited variety of problem solving techniques. McLeod (1990) theorized that dealing with emotions uses up the limited processing capability available for developing new problem solving strategies. Giangrasso also found some evidence that mathematically anxious students were less likely to approach problems systematically and tended to abandon problems quickly. Finally, McKeachie (1988) reported that highly anxious individuals were less inclined to take risks or vary behavior.

Negative correlations between mathematics anxiety and mathematics achievement are common (Adams & Holcomb, 1986; Berenson et al., 1992; Clute, 1984; Coleman, 1991; Cooper & Robinson, 1989; Covington & Omelich, 1987; Frary & Ling, 1983; Gliner, 1987; Hembree, 1990; Lawson, 1993; McCoy, 1992; Richardson & Suinn, 1972). In their study involving over one thousand college students, Resnick et al. (1982) found that students enrolled in the lower level math sequences had higher mathematics anxiety, with students in the most advanced course having lower math anxiety scores than any other group. Similarly, Betz's (1978) study involving 652 college students reported a significant negative relationship between level of math course and math anxiety level.

Because a certain degree of arousal may be necessary for performing a task, the relationship between anxiety and achievement is not necessarily linear (Bessant, 1995; Cronbach & Snow, 1977; Meece et al., 1990; Sieber et al., 1977). Under this conjecture, the relationship between anxiety and achievement would be modeled by an inverted-U, with moderately anxious students performing better than students with very low or very high anxiety toward a task.

Currently under debate are two causal models that attempt to explain the relationship between prior mathematics achievement, mathematics anxiety, mathematics avoidance, and performance (achievement) in mathematics. The anxiety-blockage model theorizes that anxiety hinders test performance by temporarily blocking previously learned mathematics. In other words, anxiety inhibits the recall of prior learning (Birenbaum & Gutvirtz, 1993; Covington & Omelich, 1987; Hembree, 1990; Naveh-Benjamin et al., 1987; Tobias, 1990). On the other hand, the deficit model postulates that deficiencies in prior knowledge cause mathematics anxiety and poor performance in future mathematics endeavors (Birenbaum & Gutvirtz, 1993; Frary & Ling, 1983; Fulkerson et al., 1984; McKeachie, 1988; Richardson & Woolfolk, 1980). Subscribers to this theory believe that anxiety hindered the original learning rather than performance on the present test. According to Cooper and Robinson (1989), there is a cycle in which a poor mathematics background leads to mathematics anxiety, which, in turn, leads to avoidance of mathematics.

In their review of ATI research in science education, Koran and Koran (1984) referred to task organization as a manipulation likely to have an obvious effect on learning and a clear implication for ATI research. That is, material that is well organized should result in better achievement for high anxiety students (Koran & Koran, 1984). Similarly, Tobias (1989) and Bessant (1995) recommended clearly structured instruction as beneficial to highly anxious students. According to Sieber et al. (1977), students high in anxiety may also benefit from opportunities for repetition of selected parts of the content.

Prior Mathematics Achievement

Prior mathematics achievement and subsequent achievement in mathematics are positively correlated (Berenson et al., 1992; Cooper & Robinson, 1989; Covington & Omelich, 1987; Frary & Ling, 1983; Goolsby et al., 1988; National Research Council, 1989; Richardson & Woolfolk, 1980; Suciu, 1991; Waits & Demana, 1988). In an analysis of previous research, Cooper and Robinson (1989) found that prior mathematics achievement accounted for about a third of the variability in test scores. In addition, a study by Dennison, Bruning, and Schraw (1995) reported that knowledge in a specific domain enabled learners to better monitor their own performance.

Over the past several years, colleges have enrolled increasing numbers of unprepared freshmen in remedial mathematics courses

(Berenson et al., 1992; National Commission on Excellence in Education, 1983). The National Research Council (1989) reported that 60% of college mathematics enrollments are in courses ordinarily taught in high school. Despite the emphasis on remediation, neither educators, mathematicians, nor researchers have been successful in reversing consistent early patterns of poor achievement and failure in mathematics (National Research Council, 1989).

As the mathematical abilities of students entering college have declined, the mathematics requirement in many university degree programs has increased (Waits & Demana, 1988). Those students who were accepted by colleges and placed in remedial programs were at a high risk of dropping out (Berenson et al., 1992; Waits & Demana). In their study of five years' worth of data on 3609 students at The Ohio State University, Waits and Demana found that only 31% of students enrolled in low level remedial mathematics courses had graduated or reached senior status after four years. Over half of these students did not finish their second year of college (Waits & Demana). Johnson (1994) studied the relationship between performance in developmental mathematics and academic success and concluded that "poor performance in ... developmental mathematics greatly increases the risk of failure or attrition for students in entry-level college mathematics" (p. 2391-A).

From an ATI standpoint, Tobias (1976, 1989) hypothesized that students with lower prior achievement require more instructional support, and conversely, that as the level of prior achievement increases, less instructional support may be required.

Research Conducted at Military Academies

Military academy cadets may be viewed as a subset of American college students. A review of educational studies conducted at military academies serves to compare and contrast research involving cadets with research conducted in more traditional settings.

Hancock (1991) conducted an ATI study investigating the effects of students' conceptual levels and direct/nondirect instructional treatments on cadets' achievement and motivation. Direct teaching methods significantly benefited the achievement and motivation of low conceptual level students while nondirect instruction benefited high conceptual level students at the United States Military Academy. This finding is consistent with ATI theory (Cronbach & Snow, 1977).

Phillips (1984) studied the relationship between levels of intellectual development and 37 variables associated with the academic, military, moral-ethical, and physical development of senior

West Point cadets. He concluded that "while cadets are continuously involved in programs of both academic and military development, they are not free to give themselves completely to either" (p. 2312-A). The results of this study verify and highlight the impact of the rigorous military, ethical, and physical training on the academic achievement of military cadets.

Halloran (1982) investigated the effect of two different feedback methods for evaluating student writing with a sample of first year West Point cadets. The experimental treatment consisted of general teacher comments and a checklist of strengths and weaknesses. The results of the study indicated that the experimental technique was successful in improving cadet writing while decreasing instructor workload (Halloran, 1982).

An study conducted by Thompson, Mitchell, Coffin, and Hassett (1979) to compare homogeneous and heterogeneous grouping in core mathematics classes at the Air Force Academy found that instructor experience level had no significant effect on cadet performance in Calculus. The researchers also found that the interaction between instructor experience level and cadet aptitude was not significant.

Summary

The topic of homework has a long and varied research history as attention to the homework issue has been dependent on historical and political events. Studies that compared homework versus no homework groups produced mixed results. In addition, many of the homework researchers failed to fully describe their treatments and did not control for the effects of extraneous variables.

Spaced review has a strong conceptual basis grounded in information processing theory. But, research on distributed practice rarely moved beyond the laboratory and into the classroom. One exception is research focusing on Saxon's continuous review method of assigning mathematics homework. Still, research on Saxon's method produced mixed results. Research suggests that the distributed practice treatment may be more beneficial to low achieving students.

Achievement in mathematics is strongly related to both prior mathematics achievement and mathematics anxiety. The majority of college students are anxious about mathematics. Mathematics anxiety correlates positively with math avoidance and negatively with math background, subsequent math performance, and confidence in learning mathematics. Colleges and universities have been enrolling increasing numbers of students who are not prepared to study college-level mathematics. Early patterns of poor mathematics achievement are difficult to reverse. Although most colleges have remedial mathematics courses, very few remedial mathematics students attain a college degree.

ATI theory implies that highly anxious students would be well-served by an instructional treatment consisting of well-organized material. Similarly, students with low prior mathematics achievement should benefit from a high level of instructional support.

Only a sparse number of aptitude-treatment interaction studies have explored distributed practice as an educational treatment, yet, distributed practice homework assignments may be a way to reach students with high mathematics anxiety or low prior success in mathematics.

CHAPTER 3

METHODS

Population and Sample

The target population for this study consists of all high school and undergraduate college students enrolled in mathematics courses. The accessible population consisted of first year cadets at the United States Air Force Academy (USAFA) in the Fall semester of 1995.

The Department of Mathematical Sciences at USAFA uses Placement exam cut-off scores to place first year cadets into their first mathematics course. In addition, high school grade point averages, Scholastic Achievement Test (SAT) scores, American College Test (ACT) scores, Advanced Placement test scores, and class standings were available and were used to make decisions about placing borderline students. Students not scoring at least 50% on the Algebra/Trigonometry placement exam were placed into Precalculus. The sample for the experiment consisted of all 375 USAFA cadets enrolled in Precalculus during the 1995 fall semester. The sample represented about 28% of the first year students. Of the remaining first year students, 519 (about 39%) were placed into Calculus I, 344 (about 26%) were placed into Calculus II, and 103 (about 8%) were placed into Calculus III. Twenty students who scored at least 40% on both the Calculus I and the Algebra/Trigonometry placement exams were concurrently placed into both Precalculus and Calculus I during the Fall semester.

Semesters at the USAFA are 42 lessons long. For this experiment, the 42 Precalculus lessons were broken into four blocks of approximately equal size. An exam was given at the end of each block. At the time of the first exam (the eighth day of classes), 351 of the original 375 cadets enrolled in Precalculus remained. Enrollment was 341 at the time of the second exam (the 19th day of classes), 338 at the time of the third exam (the 29th day of classes), and 333 at the time of the fourth exam (the 41st day of classes). At the time of the final exam, 333 of the original 375 cadets remained in the course. The loss of students during the semester was attributed to natural attrition from the Air Force Academy.

A strict attendance policy ensured that very few cadets missed exams. The few cadets who did miss exams were given make-up exams which paralleled the original exam in content and point value.

In no instance did more than eight percent of the cadets miss an exam.

Due to a USAFA policy that exempts the students in the 95th through 100th percentile (i.e., the top five percent of the students) from the final exam, final exam scores were available for only 317 students. Of the 16 students who were exempted, 11 were from the treatment group and 5 were from the control group.

The USAFA has high admission standards. To qualify for admission, students must have good grades and athletic and leadership experience (Air Force Academy Admissions Office, 1995). In addition, students must be unmarried, without dependents, and between the ages of 17 and 21 (Air Force Academy Admissions Office). The mean SAT math achievement score for incoming Air Force Academy students was 660 (recomputed to reflect the 1995 recentering of the SAT) and the mean for the math portion of the ACT for incoming students was 29.3 (B. A. Branum, personal communication, September 6, 1995). The average high school grade-point average for incoming cadets was 3.85 (B. A. Branum, personal communication, September 6, 1995) and 89% of entering cadets ranked in the top fifth of their high school class (Air Force Academy Admissions Office).

The USAFA class of 1999 consisted of 1367 students, 1353 from the United States and 14 from 13 foreign countries (Lockhart, 1995). Included were 238 minority members (17%) and 219 women (16%; Lockhart). Of the United States students, 1086 (82%) were White, 56 (4%) were Black, 85 (6%) were Hispanic, 72 (6%) were Asian American, and 19 (1%) were Native American (B. A. Branum, personal communication, September 6, 1995).

Cadets completed 38 days of Basic Cadet Training (BCT) before the start of academic classes (Lockhart, 1995). BCT is a rigorous orientation program consisting of physical conditioning, military training, and field exercises (Air Force Academy Admissions Office, 1995).

All USAFA students are required to complete a sequence of core courses which includes at least two semesters of Calculus.

Instruments

Prior Mathematics Achievement

The percentage correct on the Algebra/Trigonometry placement exam was used as the measure of prior mathematics achievement. The placement exam contained 35 multiple choice items (25 algebra items and 10 trigonometry items) and was machine scored. The test was last validated for content in 1995 by faculty members of the

USAFA math placement team. The tests were found to have high predictive validity for placing students into Precalculus as their first mathematics class, with 87% of students successfully completing Precalculus with a grade of B+ or less (A's and A-'s were considered erroneously placed; W. A. Kiele, personal communication, April 5, 1995). Because the math placement exams are administered under stressful and standardized conditions, the Department of Mathematical Sciences at the USAFA considers these exams a better predictor of performance than the SAT (W. A. Kiele, personal communication, April 5, 1995). Many of the placement test items are anchored, that is, used again from year to year. The use of anchored items improves test stability and reliability and allows the faculty to compare the mathematical abilities of new cadets with those of previous classes.

The placement exams were administered under standardized conditions a few days after the students arrived at the Air Force Academy. Students took the exam in large lecture halls proctored by instructors. Standardized directions were printed on the first page of the exam and read aloud by the proctors. All students had identical time limits. The use of calculators was not permitted.

Mathematics Anxiety

Mathematics anxiety was measured by a subset of items from the Math Anxiety Rating Scale (MARS), college and adult version (Suinn, 1972). The MARS is a 98 item self-rating scale set in a five point Likert format designed as a diagnostic or screening tool for measuring mathematics anxiety. Scores on each MARS item represent the level of anxiety reported for a specific situation. Selections range from 1 representing not at all anxious to 5 representing very much anxious. An overall mathematics anxiety score is achieved by summing the individual item scores. According to Dew, Galassi, and Galassi (1983), of all of the instruments developed to measure mathematics anxiety, the MARS has the "greatest amount of psychometric, reliability, and validity data" (p. 444).

Reviews of the MARS by Richardson and Suinn (1972) and in the Mental Measurements Yearbook (Buros, 1978) attest to the internal consistency (coefficient alpha = .97) and test-retest reliability (Pearson reliability coefficients between .78 and .85) of the instrument. Richardson and Suinn (1972) obtained strong evidence of construct validity by correlating scores on the MARS with scores on the Differential Aptitude Test under the hypothesis that mathematics anxiety interferes with performance, and poor performance produces anxiety. The Pearson product-moment correlation coefficient between subjects' scores on the two instruments (r = -.64) indicates that high MARS scores are associated with poor performance on mathematics tests. Construct validity was further evidenced by reductions in MARS scores of 50 to 70 points for clients who were treated for mathematics anxiety (Buros, 1978).

Since its publication in 1972, the MARS has been the prevailing instrument for measuring mathematics anxiety (Alexander & Martray, 1989). Alexander and Martray (1989) used a two-staged process to develop an abbreviated version of the MARS. First they performed a factor analysis, reducing the 98-item MARS to 69 items by selecting the items most highly correlated to each of the five identified factors. The correlation between the 69-item MARS and the full 98-item MARS survey was high (r = .93), as was the internal consistency of the instrument (r = .97). The 69-item MARS was again abbreviated by application of factor analysis. This second factor analysis produced three principal factors: (a) mathematics test anxiety, (b) numerical task anxiety, and (c) math course anxiety (Alexander & Martray). Items that correlated highly with each of the three factors were selected for Alexander and Martray's 25-item abbreviated MARS. The 25-item MARS was shown to have high internal consistency within each of the three factors (coefficient alpha of .96, .86, and .84, respectively). In addition, correlation between the 25-item and 69-item versions of the MARS was found to be high ($\underline{r}$ = .93) and test-retest reliability after two weeks was also high ($\underline{r}$ = .86).

Alexander and Martray (1989) declared that the 25-item MARS was a "psychometrically equivalent alternative" to the 98-item MARS, while being more efficient, less costly, and easier to implement (p. 149).

The abbreviated MARS was administered to the control and treatment groups during the last ten minutes of the second lesson. When it was realized that the survey contained only the fifteen items relating to mathematics test anxiety, the 25-item survey was administered during the fifth week of class. The 25-item survey contained the fifteen items on the original survey and an additional ten items relating to numerical task anxiety and math course anxiety. Additional analyses were planned to compare the results of the two surveys. On both administrations of the survey, a standardized set of instructions was read aloud by the instructors. Students were assured that their instructors would not have access to the individual MARS scores. The surveys were machine scored. Scores obtained from the 25-item surveys were used as the measure of mathematics anxiety.

Precalculus Achievement

Six variables were used to measure student achievement in Precalculus. Included were four hourly exams, a final exam, and the final course percentage grade. (See Appendix A for hourly exams. Test security prohibited the inclusion of the final exam.)

All exam items were written by members of the USAFA Department of Mathematical Sciences and the same exam was administered to all students. Parallel make-up exams were administered to the few students who missed an exam. The hourly exams were composed of multiple choice and open-ended items. The final exam consisted of mostly multiple-choice items (69%) with several open-ended items (31%). The exams were reviewed by several mathematics instructors for content validity. Split-half reliability coefficients for all exams were calculated using the Spearman-Brown prophecy formula (see Table 1) and were found to be acceptable (Fraenkel & Wallen, 1993).

As standard procedure at the Air Force Academy, exams were administered to the entire course population during the same period of time. Hourly exams were given from 7:00 to 7:50 a.m., before the start of classes. Students were assigned to lecture halls and classrooms. Standardized directions were printed on the first page of

Table 1

Split-Half Reliability Coefficients for Precalculus Exams

Exam	Number of items	Reliability coefficient
I	16	.71
II	13	.77
III	13	.69
IV	14	.68
Final	81	.83

the exams and were read aloud by the instructors administering the exam. All students had identical time limits. Students in both the treatment and control groups were permitted to use calculators on all four hourly exams.

The final exam was given seven days after the last day of class and was administered in two parts. Students were given 1 hour to complete Part I of the exam and 2 hours and 50 minutes to complete Part II. With the exception of five items, Part I was identical to the Algebra/Trigonometry Placement Exam. Part II was a cumulative exam containing mostly anchored items. Students were not permitted to use calculators on Part I of the final exam. The use of calculators was permitted on Part II.

The multiple choice exam items were machine scored. Standardized rubrics were used to score open-ended items. For the exam items that were scored by more than one instructor, a sample of 30 exams (15 from the treatment group and 15 from the control group) was selected for duplicate scoring. Inter-scorer reliability was calculated and found to be high (correlation coefficients of .99, .98, .87, and .96). All exam scores were converted to percentages.

The final course percentage grade was based on the following sub-scores: (a) four hourly exams, 45%; (b) final exam, 30%; (c) three written exercises, 5%; (d) course project, 5%; (e) three group problem solving exercises, 5%; and (f) quiz, homework, and participation points awarded by the individual instructors, 10%.

Procedures

This experiment employed the ATI compensatory instructional model. The distributed practice treatment was designed to interact with the low prior achievement and high mathematics anxiety student aptitudes by circumventing or neutralizing their debilitating effects (Salomon, 1972).

As recommended in previous ATI and homework research, the duration of the treatment was one semester, the entire duration of the Precalculus course (Austin, 1979; Becker, 1970; Becker & Young, 1978; Cronbach & Snow, 1977; Holtan, 1982; Koran & Koran, 1984; Snow, 1977a).

Although assignment to Precalculus sections was not purely random, student course schedules at the USAFA are computer generated and students (especially first year students) have very few choices in their schedules. The treatment group consisted of approximately 46% of the Precalculus students (161 students divided into eight sections). The control group consisted of the remaining students enrolled in Precalculus (190 students divided into nine sections).

To minimize instructor workload, each instructor was assigned either all treatment sections or all control sections. The Precalculus sections were taught by eight different instructors; three instructors taught treatment group sections and five instructors taught control group sections. Four of the control group sections experienced an instructor change mid-semester because one instructor was on medical leave during the first half of the semester and another attended military training during the second half of the semester.

All instructors were active duty members of the United States Air Force. Degree levels for instructors ranged from bachelor to doctoral with most instructors holding a master of science degree (see Table 2). Undergraduate degree specialties for the instructors included operations research, engineering mechanics, mathematics, and electrical engineering. Graduate specialties included industrial engineering, system engineering, business administration, mathematics, operations research, and theology. Instructor experience level varied from first year instructors to a seasoned instructor with over 20 years teaching experience (see Table 2). Although most of the instructors had some prior teaching experience, few had prior experience teaching Precalculus (see Table 2). Both experienced and

Table 2

Assignment of Instructors to the Treatment and Control Groups

Instructor	Number of sections	Treatment or Control	Highest academic degree	Years teaching math	Semesters teaching Precalculus
D	1	Control	MS[b]	0.5	1
S	3	Control	MS	2	0
W/B[a]	3	Control	MS/MS	2/1	0/1
P	1	Control	MS	11	0
W/L[a]	1	Control	MS/BS[c]	2/0	0/0
F	4	Treatment	MS	0	0
L	3	Treatment	BS	0	0
C	1	Treatment	PhD[d]	20.5	3
Control group means				2.56[e] years	0.28[e] semesters
Treatment group means				2.56[e] years	0.38[e] semesters

[a]Mid-semester instructor change.
[b]Master of Science.
[c]Bachelor of Science.
[d]Doctoral.
[e]Means weighted by number of sections.

inexperienced instructors were assigned to each group in an attempt to equalize instructor experience across groups (see Table 2).

The course topics, textbook, handouts, reading assignments, and graded assignments (with the exception of quiz, homework, and participation points) were identical for the treatment and control groups. The listing of homework assignments in the syllabus differed between groups (see Appendix B).

The control group was assigned daily homework related to the topic(s) presented that day in class. Peterson (1971) calls this the vertical model for assigning mathematics homework. The treatment group was assigned homework in accordance with a distributed organizational pattern that combines practice on current topics and reinforcement of previously covered topics (Peterson, 1971; Hirsch et al., 1982). Under the distributed model, approximately 40% of the problems on a given topic were assigned the first day, with an additional 20% assigned on the next lesson and the remaining 40% of problems on the topic appearing on subsequent assignments. In Hirsch's research and in this study, after the initial homework assignment, a problem representing a given topic resurfaced on the 2nd, 4th, 7th, 12th, and 21st lesson. Consequently, treatment group homework for lesson one consisted of only one topic; homework for lessons two and three consisted of two topics; and homework for lesson four through six consisted of three topics. This pattern continued as new topics were added and was applied to all non-exam, non-laboratory lessons. Hirsch et al. (1982) described the distributive organizational pattern as "the easiest of the non-conventional patterns to implement in the mathematics classroom" (p. 245).

Table 3 shows the traditional and distributed practice patterns that were used to assign homework. The same homework problems were assigned to both groups with only the pattern of assignment differing (see Tables 4 and 5).

Because of the nature of the distributed practice model, homework for the treatment group contained fewer problems (relative to the control group) early in the semester with the number of problems increasing as the semester progressed. Later in the semester, homework for the treatment group contained more problems (relative to the control group). As shown in Tables 4 and 5, by the end of the semester, both groups had been assigned precisely the same homework problems.

Table 3

Distributed Homework Model.

Day	Distributed practice topic(s)	Traditional topic	Day	Distributed practice topic(s)	Traditional topic
1	A	A	16	EJMOP	P
2	AB	B	17	FKNPQ	Q
3	BC	C	18	GLOQR	R
4	ACD	D	19	HMPRS	S
5	BDE	E	20	INQST[a]	T
6	CEF	F	21	AJORU	U
7	ADFG	G	22	BKPSUV	V
8	BEGH	H	23	CLQVW	W
9	CFHI	I	24	DMRUWX	X
10	DGIJ	J	25	ENSVXY	Y
11	EHJK	K	26	FOWYZ	Z
12	AFIKL	L	27	GPUXZa[b]	a
13	BGJLM	M	28	HQVYab	b
14	CHKMN	N	29	IRWZbc	c
15	DILNO	O	30	JSXacd	d

Note. From "Alternative Models for Mathematics Assignments," by C. R. Hirsch, S. F. Kapoor, and R. A. Laing, 1982, International Journal of Mathematical Education in Science and Technology, 13, p. 246. Adapted with permission (see Appendix D).

[a]Homework on topic "T" was not distributed due to a late syllabus change).

[b]Upper and lower case letters represent different topics.

Table 4

Homework Problems Assigned to the Control Group

Lesson											# of Problems
2	A1	A2	A3	A4	A5	A6	A7	A8			8
3	B1	B2	B3	B4	B5	B6	B7	B8	B9		9
4	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	10
5	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	10
6	E1	E2	E3	E4	E5	E6	E7	E8			8
7	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	10
8	G1	G2	G3	G4	G5	G6	G7	G8			8
Exam I											63
10	H1	H2	H3	H4	H5	H6	H7	H8			8
11	I1	I2	I3	I4	I5	I6	I7	I8	I9		9
13	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	10
14	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	10
15	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	10
17	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	10
18	N1	N2	N3	N4	N5	N6	N7	N8			8
Exam II											65
21	O1	O2	O3	O4	O5	O6	O7	O8	O9		9
22	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	10
23	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	10
24	R1	R2	R3	R4	R5	R6	R7	R8	R9		9
25	S1	S2	S3	S4	S5	S6	S7	S8	S9		9
27	T1	T2	T3	T4	T5	T6					6
28	U1	U2	U3	U4	U5	U6	U7	U8			8
Exam III											61
31	V1	V2	V3	V4	V5	V6	V7	V8			8
32	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	10
33	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	10
34	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8			8
35	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9		9
36	a1[a]	a2	a3	a4	a5	a6	a7	a8			8
37	b1	b2	b3	b4	b5	b6	b7	b8	b9		9
38	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	10
39	d1	d2	d3	d4	d5	d6	d7	d8			8
Exam IV											80
Total											269

Note. A1 represents the first problem in topic A, A2 represents the second problem, etc.
[a]Upper and lower case letters represent different topics.

Table 5

Homework Problems Assigned to the Experimental Group

Lesson																# Problems
2	A1	A2	A3													3
3	A4	B1	B2	B3												4
4	B4	B5	C1	C2	C3	C4										6
5	A5	C5	C6	D1	D2	D3	D4									7
6	B6	D5	D6	E1	E2	E3										6
7	C7	E4	F1	F2	F3	F4										6
8	A6	D7	F5	F6	G1	G2	G3									7
Exam I																39
10	B7	E5	G4	H1	H2	H3										6
11	C8	F7	H4	I1	I2	I3										6
13	D8	G5	I4	I5	J1	J2	J3	J4								8
14	E6	H5	J5	J6	K1	K2	K3	K4								8
15	A7	F8	I6	K5	K6	L1	L2	L3	L4							9
17	B8	G6	J7	L5	L6	M1	M2	M3	M4	M5						10
18	C9	H6	K7	M6	M7	N1	N2	N3	N4							9
Exam II																56
21	D9	I7	L7	L8	N5	O1	O2	O3	O3							9
22	E7	J8	M8	O4	O5	O6	P1	P2	P3	P4						10
23	F9	K8	N6	P5	P6	P7	Q1	Q2	Q3	Q4						10
24	G7	L9	O7	Q5	Q6	Q7	R1	R2	R3							9
25	H7	M9	P8	R4	R5	R6	S1	S2	S3	S4						10
27	I8	N7	Q8	S5	S6	T1[a]	T2[a]	T3[a]	T4[a]	T5[a]	T6[a]					11
28	A8	J9	O8	R7	U1	U2	U3	U4								8
Exam III																67
31	B9	K9	P9	S7	U5	U6	V1	V2	V3	V4	V5					11
32	C10	L10	Q9	V6	W1	W2	W3	W4	W5	W6						10
33	D10	M10	R8	U7	W7	W8	X1	X2	X3	X4	X5	X6				12
34	E8	N8	S8	V7	X7	X8	Y1	Y2	Y3	Y4	Y5					11
35	F10	O9	W9	Y6	Y7	Z1	Z2	Z3	Z4	Z5						10
36	G8	P10	U8	X9	Z6	Z7	Z8	a1[b]	a2	a3	a4					11
37	H8	Q10	V8	Y8	a5	a6	a7	b1	b2	b3	b4	b5	b6			13
38	I9	R9	W10	Z9	b7	b8	b9	c1	c2	c3	c4	c5	c6	c7		14
39	J10	S9	X10	a8	c8	c9	c10	d1	d2	d3	d4	d5	d6	d7	d8	15
Exam IV																107
Total																269

Note. A1 represents the first problem in topic A, A2 represents the second problem, etc.

[a]Homework on topic "T" was not distributed due to a late syllabus change.

[b]Upper and lower case letters represent different topics.

In addition to the homework described in tables 3, 4, and 5, both groups were directed to prepare for class by reading the appropriate textbook pages (Algebra and Trigonometry by Keedy, Bittenger, & Beecher, 1993) and working the margin exercises. The margin exercises are designed to provide practice and exploration for topics to be covered during the upcoming lesson. Homework assignments for the control and treatment groups were made from the Algebra and Trigonometry (Keedy et al.) textbook and are listed in the course syllabus (see Appendix B).

Because homework was the key manipulated variable in this experiment, and because larger effects on achievement were sometimes found when homework was graded (Austin, 1979; Lai, 1994; Paschal et al., 1984), instructors were directed to collect all homework. Collected homework was checked and coded for correctness and completion on a three point scale (0 = less than one-third complete and correct, 1 = one-third to two-thirds complete and correct, and 2 = more than two-thirds complete and correct). This grading scheme is consistent with homework grading practices in first year mathematics courses at the USAFA.

In addition to overall correctness/completion codes, the treatment group's homework was coded for correctness and completion under two categories: (a) new topics, and (b) review topics(s), using the same three point coding system. Furthermore, a sample of control and treatment group homework was selected for a more detailed problem by problem error analysis in three key topic areas: (a) functions; (b) equations and inequalities with absolute value; and (c) trigonometric functions. Of all the homework analyses, only the overall homework correctness/completion scores were provided to instructors.

Instructors in both groups were encouraged to use class time to discuss and review the assigned homework problems. Prior to the second, third, and fourth exam, and at the end of the semester, both groups spent one lesson in review. Review lessons were planned by the individual instructors.

Treatment Verification

As recommended by Shaver (1983), several methods of treatment verification were employed: (a) classroom observation; (b) instructor surveys (mid-semester and end of semester); (c) student surveys (mid-semester and end of semester); and (d) homework completion logs. The student surveys were piloted using groups of students from a community college and a private university in Florida.

(See Appendix C for sample classroom observation log and student and instructor surveys.)

The Department of Mathematical Sciences at the Air Force Academy routinely collects study time data. After each exam, a large sample of cadets anonymously report the amount of time (in minutes) spent preparing for each class up to and including the exam. These data were available as an additional source of treatment verification.

Data Analysis

Testing for Group Parity

Although the section assignment process should have ensured that the groups were approximately equal in prior mathematics achievement and mathematics anxiety, group equality was tested before testing for a main treatment effect.

Analysis of Anxiety Data

Students took two math anxiety surveys. The first survey contained the 15 items identified by Alexander and Martray (1989) as relating only to mathematics test anxiety. The second survey contained all 25 items from the abbreviated MARS (Alexander & Martray, 1989). The students responded to the 15-item survey on the second day of class and the 25-item survey five weeks later. The two surveys had 15 items in common. Mathematics test anxiety sub-scores were calculated using the 15 relevant items from the 25-item survey. These sub-scores were compared (via correlation) with the scores calculated from the first survey. In addition, item-by-item correlations were calculated for the 15 common items.

Scores from the 25-item survey were used as the measure of mathematics anxiety in all subsequent analyses.

Cronbach and Snow (1977) warned that the relationship between anxiety and achievement is not necessarily linear. For this reason, an analysis of the relationship between prior mathematics achievement and mathematics anxiety was conducted. Hierarchical multiple regression analysis using power polynomials (Cohen & Cohen, 1983) was employed to determine whether the effect of mathematics anxiety on prior achievement was best modeled by a linear, quadratic, or cubic anxiety variable.

Hypothesis Testing

Hierarchical multiple regression was also used to test the hypotheses. Four sets of independent variables were defined. Set A, the covariate set, contained two variables: (a) prior math achievement, and (b) mathematics anxiety. Set B contained the variable representing group membership (treatment group or control group). Set C, the two-way interaction set, contained two interaction variables: (a) Prior Achievement × Treatment, and (b) Math Anxiety × Treatment. Set D contained the variable representing the three-way interaction between prior mathematics achievement, math anxiety, and the distributed practice treatment.

The dependent variable in this study was Precalculus achievement. Precalculus achievement was measured as the semester progressed and produced six scores: four hourly exam scores, a final exam score, and a final course percentage grade.

The independent variable sets were added one at a time via hierarchical multiple regression. The procedure was repeated for each of the six measures of Precalculus achievement. By analyzing each measure of achievement separately, the goal was to determine whether the length of treatment had an impact on achievement with the expectation that the distributed practice treatment would have a cumulative effect (Austin, 1979).

By performing so many non-independent tests, experimentwise error was multiplied; that is, a certain percentage of true null hypotheses may have been rejected due to error. To control for this threat, achievement measured by the final course percentage grade was tested first, with the other scores (hourly exams and final exam) tested only if the regression equation predicting the final course grade produced a statistically significant $\underline{F}$-ratio.

Testing for a Main Effect

The first two steps of the hierarchical regression tested for a main treatment effect. The covariate variables (Set A) were entered in the first step and the group membership variable (Set B) was entered in the second step. $\underline{F}$-tests of the semi-partial correlation coefficients were conducted to determine whether the treatment made a significant contribution to achievement variance after controlling for the covariates (prior achievement and math anxiety).

Testing for Two-Way ATI

The next step in the hierarchical regression analysis was to add Set C, the two-way interaction variables. Again, $\underline{F}$-tests of the semi-partial correlation coefficients were conducted. If the interaction variable was found to contribute a significant proportion of variance to Precalculus achievement, separate regression equations would be calculated for each group (treatment and control). The point of intersection would then be determined and the Johnson-Neyman technique would be employed to determine the region of significance for the interaction (Bracht, 1970; Pedhazur, 1982).

Testing for Three-Way ATI

The analysis of the three-way interaction is important because, according to Cronbach and Snow (1977), "power is lost if the analysis of Anxiety × Treatment interaction does not take ability into account" (p. 399). Cronbach and Snow recommend the inclusion of an Ability × Anxiety × Treatment term in the regression equation. To analyze the three-way interaction, Set D was added in the final step of the hierarchical regression. Again, F-tests of the semi-partial correlation coefficients were conducted to determine whether the three-way interaction accounted for a significant proportion of variance in Precalculus achievement when all of the other variables had been controlled for.

Testing for Instructor Effect

Hierarchical multiple regression analysis was also used to determine whether there was a significant effect due to the instructor. Two variable sets were defined. Set A′ consisted of the covariates (prior achievement and mathematics anxiety scores) and the group membership variable (treatment group or control group). Set B′ included the dummy-coded instructor variables (Cohen & Cohen, 1983). The first step of the hierarchical regression included only Set A′ with Set B′ added in the second step. The F-ratio of the semi-partial correlation coefficient was tested to determine whether there was an instructor effect beyond what was already explained by prior achievement, math anxiety, and the distributed practice treatment. This procedure was repeated for each of the six measures of Precalculus Achievement.

Analysis of Treatment Verification Data

Treatment verification data were collected from five sources: (a) student surveys (mid-semester and end of semester); (b) instructor surveys (mid-semester and end of semester); (c) observations; (d) homework completion logs; and (e) study time surveys.

Both student surveys contained two questions pertaining to the homework assignment models. A t-test of the means was employed to determine whether the treatment and control groups differed in their descriptions of the homework model. Both surveys also asked questions pertaining to the collection and distribution of homework. Another survey item asked students to report potential treatment crossover.

Items on the instructor surveys also served to verify the treatment. Instructors were asked if they followed the syllabus and if they discussed homework problems in class.

Classroom observations were used to verify that the instructors followed the syllabus for their particular group. Homework distribution, collection, and review procedures were also documented during the observations.

Study-time data provided an additional source of treatment verification. Statistical t-tests of the means were conducted to determine whether there was a between groups difference in time spent completing homework and studying for exams.

Other Analyses

To determine whether homework scores differed between the groups, t-tests were conducted using the homework completion/correctness scores.

Statistical t-tests were conducted on the treatment group homework scores to determine whether there was a difference between scores based on new topics and scores based on review topics.

Additionally, homework assignments from two control group sections and two treatment group sections were subjected to a detailed error analysis in three key topic areas. The purpose of this analysis was to determine whether the distributed practice treatment allowed the treatment group to correct their misconceptions (Ausubel, 1966).

Homework scores and exam scores were correlated to determine the effect of homework completion on exam performance.

Alpha, Power, and Effect Size

Snow (1977b) recommended large-scale ATI studies so that powerful statistical tests could be employed. In this study, the sample size was set by course enrollment. Alpha was set at .05 and a small to medium effect size ($f^2 = .11$) was postulated. The selection of effect size was based on previous research results (Revak, 1994; Hirsch et al., 1982, 1983) and Cohen's (1969) belief that "the state of development of much behavioral science is such that not very much variance in the dependent variable is predictable" (p. 75). Tables and formulas presented by Cohen and Cohen (1983) and Cohen were used to determine the power of the statistical tests.

All tests of the main treatment effect had at least 90% power to detect a small to medium effect size at an alpha level of .05. Tests of the two-way aptitude-treatment interactions had at least 85% power and tests of the three-way ATIs had at least 80% power. Tests of the instructor effect had at least 75% power in all cases.

The hierarchical regressions used to test the linearity of the relationship between prior achievement and anxiety all had an associated power greater than 99%.

Statistical *t*-tests were used to make certain comparisons between the control and treatment groups (i.e., study time, homework scores, and responses to survey items). All *t*-tests had at least 85% power to detect a medium effect size (d = 0.5) at an alpha level of .05.

All estimates of power are conservative because calculations were based on the sample size of the (smaller) treatment group.

CHAPTER 4

RESULTS

Descriptive Statistics

The means and standard deviations for the entire sample and for the treatment and control groups on measures of prior achievement, mathematics anxiety, and Precalculus achievement are reported in Table 6.

Group Parity

Prior Achievement

A $\underline{t}$-test of the means on the Algebra/Trigonometry placement test confirmed that treatment and control groups were not significantly different in prior mathematics achievement, $\underline{t}(349) = 1.23$, $\underline{p} = .22$.

Mathematics Anxiety

Two measures of mathematics anxiety were conducted. A 15-item survey of mathematics test anxiety was administered on the second day of class. The 25-item abbreviated MARS (Alexander & Martray, 1989) was administered on the 12th day of class. Analysis of data from the 15-item mathematics anxiety survey confirmed that the treatment and control groups were not significantly different in their mathematics test anxiety, $t(349) = 1.20$, $p = .23$, whereas analysis of data from the 25-item survey indicated that the treatment group was less anxious about mathematics than the control group, $t(349) = 2.44$, $p = .02$.

Table 6

Descriptive Statistics for Measures of Prior Achievement, Anxiety, and Precalculus Achievement

	Prior achievement	Math anxiety 15-item	Math anxiety 25-item	1st Exam	2nd Exam	3rd Exam	4th Exam	Final Exam	Final course grade
All students									
N	351	351	351	351	341	338	333	317	333
M	35.88	40.58	51.51	80.43	70.67	70.48	65.21	70.43	74.83
SD	8.74	11.90	14.44	13.25	13.67	13.10	13.55	11.13	8.55
min	5.00	15.00	28.00	14.81	21.48	29.63	23.70	20.33	35.00
max	50.00	74.00	99.00	99.26	96.30	100.00	100.00	94.67	96.76
Treatment group									
n	161	161	161	161	160	157	155	144	155
M	36.51	39.74	49.48	82.69	73.58	70.71	68.28	71.7	76.96
SD	8.09	11.77	12.96	11.89	12.79	12.99	12.73	10.60	7.84
min	5.00	15.00	28.00	28.99	37.78	29.63	23.70	28.61	46.43
max	50.00	74.00	93.00	99.26	95.56	98.52	100.00	93.56	94.83
Control group									
n	190	190	190	190	181	181	178	173	178
M	35.36	41.28	53.23	78.51	68.10	70.27	62.54	69.41	72.97
SD	9.24	12.00	15.42	14.05	13.93	13.23	13.71	11.48	8.72
min	5.00	15.00	28.00	14.81	21.48	30.37	28.15	20.33	35.00
max	47.50	69.00	99.00	99.26	96.30	100.00	99.26	94.67	96.76

Note. All prior achievement and achievement scores are measured in percent

Analysis of Anxiety Survey Data

Because both measures of mathematics anxiety had 15 items in common, analyses were conducted to compare student responses on the common items. The 15 relevant items from the 25-item MARS were used to calculate a mathematics test anxiety sub-score. This sub-score was compared (via correlation) to the mathematics test anxiety score obtained from the 15-item survey. A medium-high positive correlation between the two was found ($r = .76$). Item-by-item correlations produced modest positive correlations (see Table 7). A t-test of the means revealed that the students reported significantly less mathematics test anxiety on the second administration of the survey , $t(350) = 4.89$, $p = 0$.

The relationship between math anxiety and prior math achievement was analyzed to determine whether mathematics anxiety was best represented by a linear, quadratic, or cubic model. Hierarchical multiple regression using power polynomials (Cohen & Cohen, 1983) disclosed that the relationship between mathematics anxiety and prior mathematics achievement could best be modeled linearly. The addition of a quadratic or cubic term did not account for any variance in prior achievement beyond what had already been accounted for by the linear anxiety term (see Table 8).

Table 7

Item-by-Item Correlations on the 15 Items Common to Both Surveys of Mathematics Anxiety

Anxiety survey item	Correlation (r)
Studying for a math test.	.53
Taking the math section of a college entrance exam.	.60
Taking an examination (quiz) in a math course.	.46
Taking an examination (final) in a math course.	.59
Picking up the math textbook to begin working on a homework assignment.	.52
Being given a homework assignment of many difficult problems which is due the next class meeting.	.48
Thinking about an upcoming math test one week before.	.56
Thinking about an upcoming math test one day before.	.61
Thinking about an upcoming math test one hour before.	.61
Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	.55
Picking up a math textbook to begin a difficult reading assignment.	.47
Receiving your final math grade in the mail.	.63
Opening a math or stat book and seeing a page full of problems.	.48
Getting ready to study for a math test.	.43
Being given a "pop" quiz in a math class.	.57

Table 8

Analysis of Mathematics Anxiety Variable for Curvilinearity

Independent variables	Cumulative R^2	df	F	Variables added	Increment to R^2	df	F of the increment
x_1	.023	1,349	8.19**				
x_1, x_1^2	.031	2,348	5.52**	x_1^2	.008	1,348	2.81
x_1, x_1^2, x_1^3	.031	3,347	3.67*	x_1^2, x_1^3	0	1,347	0

$^*p < .05$. $^{**}p < .01$

Hypothesis Testing

Hierarchical multiple regression was employed to test the hypotheses. The four sets of independent variables are described in Table 9. Table 10 shows the results of the step-by-step hierarchical regressions as the sets of independent variables were added to the regression equations.

Table 9

Variables for Hierarchical Data Analysis

Set	Description	Independent variables
A	Covariates	x_1 = Prior achievement x_2 = Math anxiety
B	Group membership	x_3 = Treatment/Control
C	2-way interactions	$x_4 = x_1 \times x_3$ $x_5 = x_2 \times x_3$
D	3-way interaction	$x_6 = x_1 \times x_2 \times x_3$

Effect of the Covariates

Step one of the hierarchical analyses tested the effect of the covariates (prior mathematics achievement and mathematics anxiety) on Precalculus achievement. Set A was regressed on each of the six measures of Precalculus achievement. Regression analysis revealed that a significant proportion of variance in all six measures of

Precalculus achievement was explained by the covariate set (see Table 10).

Main Treatment Effect

Step two of the hierarchical analyses tested for a main effect due to the distributed practice treatment. The covariates (Set A) and the group membership variable (Set B) were regressed on each of the six measures of Precalculus achievement. Tests of the semi-partial correlation coefficients revealed that, when the covariates were controlled for, the distributed practice treatment accounted for a significant proportion of the variance in Precalculus achievement in all but the third exam and final exam (see Table 10).

Two-Way ATI Effects

Step three of the hierarchical regression analysis added the two aptitude-treatment interaction variables (Set C). The semi-partial correlation coefficients were tested to determine whether the two-way interactions accounted for any variance in Precalculus achievement above what had already been accounted for by prior achievement, anxiety, and the main effect. The effect of the two-way ATIs was not statistically significant for any of the six measures of Precalculus achievement (see Table 10).

Table 10

Hierarchical Multiple Regression Analysis

Independent variable sets	Cumulative R^2	df	F	Variable sets added	Increment to R^2	df	F of the increment
First exam							
A	.239	2, 348	54.66***	A			
A, B	.249	3, 347	38.41***	B	.010	1, 347	4.73*
A, B, C	.251	5, 345	23.07***	C	.001	2, 345	0.30
A, B, C, D	.253	6, 344	19.41***	D	.002	1, 344	1.05
Second exam							
A	.169	2, 338	34.42***	A			
A, B	.193	3, 337	26.80***	B	.023	1, 337	9.78**
A, B, C	.198	5, 335	16.49***	C	.005	2, 335	1.07
A, B, C, D	.198	6, 334	13.77***	D	.001	1, 334	0.33
Third exam							
A	.069	2, 335	12.46***	A			
A, B	.070	3, 334	8.34***	B	.000	1, 334	0.13
A, B, C	.073	5, 332	5.21***	C	.003	2, 332	0.56
A, B, C, D	.074	6, 331	4.43***	D	.002	1, 331	0.56
Fourth exam							
A	.093	2, 330	16.97***	A			
A, B	.124	3, 329	15.53***	B	.031	1, 329	11.57**
A, B, C	.128	5, 327	9.59***	C	.004	2, 327	0.71
A, B, C, D	.128	6, 326	7.97***	D	.000	1, 326	0.00
Final exam							
A	.121	2, 314	21.61***	A			
A, B	.125	3, 313	14.84***	B	.004	1,313	1.27
A, B, C	.126	5, 311	8.96***	C	.001	2,311	0.24
A, B, C, D	.127	6, 310	7.49***	D	.001	1,310	0.24
Final course percentage grade							
A	.203	2, 330	41.90***	A			
A, B	.234	3, 329	33.49***	B	.031	1, 329	13.48**
A, B, C	.236	5, 327	20.16***	C	.002	2, 327	0.36
A, B, C, D	.236	6, 326	16.81***	D	.001	1, 326	0.27

*p < .05 **p < .01 ***p < .001

Three-Way ATI Effect

The three-way aptitude-treatment interaction between the distributed practice treatment, prior mathematics achievement, and mathematics anxiety (Set D), was entered in the final step of the hierarchical multiple regression. Analysis of the semi-partial correlation coefficients revealed that the three-way interaction did not contribute to the variance in Precalculus achievement beyond what was already explained by the covariates, the treatment, and the two-way ATIs (see Table 10).

Instructor Effects

Regression analysis was used to determine whether there was a significant effect due to instructor after prior achievement, anxiety, and the treatment were controlled for. A two-step hierarchical regression was employed with the covariate and group membership variables (Set A′) entered in the first step and the dummy-coded instructor variable set (Set B′) added in the second step. Semi-partial correlation coefficients were calculated and F-tests were conducted. This analysis revealed that the instructors did not contribute to the variance in Precalculus achievement beyond what was already accounted for by prior achievement, anxiety, and group membership (see Table 11).

Table 11

Analysis of Instructor Effects

Independent variable sets	Cumulative R^2	df	F	Variable sets added	Increment to R^2	df	F of the increment
First exam							
A′	.249	3, 347	38.41***	A′			
A′, B′	.268	10, 340	12.46***	B′	.019	7, 340	1.25
Second exam							
A′	.193	3, 337	26.80***	A′			
A′, B′	.208	10, 330	8.66***	B′	.015	7, 330	0.91
Third exam							
A′	.070	3, 334	8.34***	A′			
A′, B′	.098	10, 327	3.55**	B′	.028	7, 327	1.45
Fourth exam							
A′	.124	3, 329	15.53***	A′			
A′, B′	.133	10, 322	4.95***	B′	.009	7, 322	0.50
Final exam							
A′	.125	3, 313	14.84***	A′			
A′, B′	.141	10, 306	5.01***	B′	.016	7, 306	0.82
Final course percentage grade							
A′	.234	3, 329	33.62***	A′			
A′, B′	.239	10, 322	10.12***	B′	.004	7, 322	0.21

Note. Set A′ = covariates and group membership variable; Set B′ = instructors.
***p < .001

Treatment Verification

Several methods of treatment verification were employed to ensure that the treatment was administered as planned and directed.

Analyses of items on student surveys indicated that the control group and the treatment group did receive different types of homework assignments (see Table 12).

Table 12

Responses to Survey Items Concerning the Composition of Homework Assignments

Mid-Semester survey item	Treatment mean	Control mean	df	t
Daily homework problems relate only to topics covered in the most current lesson.	2.28	3.60	316	10.78***
Daily homework problems consist of problems related to the current lesson mixed with problems related to topics covered in previous lessons.	4.31	3.20	316	10.56***
End of semester survey item	**Treatment mean**	**Control mean**	**df**	**t**
Daily homework problems were related only to topics covered in the most current lesson.	2.28	3.86	322	12.48***
Daily homework problems consisted of problems related to the current lesson mixed with problems related to topics covered in previous lessons.	4.15	2.60	318	11.99***

Note. the scale used for these survey items was:
1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree,
5 = strongly agree.
$^{***}p < .001$

Student surveys also confirmed that homework was collected and checked for correctness and completion. In addition, student surveys indicated that homework was returned promptly, and that the distributed practice treatment was confined to the treatment group (see Table 13).

Table 13

Responses to Survey Items Concerning Homework Collection and Potential Treatment Crossover

Mid-Semester survey item	Mean for all students	Treatment mean	Control mean
Homework is collected and checked for correctness and completion.	4.80	4.79	4.80
Homework is returned promptly.	4.53	4.54	4.53
I work on homework with students who use a different homework syllabus.	1.47	1.36	1.56
End of semester survey item	**Mean for all students**	**Treatment mean**	**Control mean**
Homework was collected and checked for correctness and completion.	4.66	4.67	4.65
Homework was returned promptly.	4.54	4.51	4.58
I worked on homework with students who used a different homework syllabus.	1.84	1.95	1.72

Note. the scale used for these survey items was: 1 = never, 2 = rarely, 3 = sometimes, 4 = frequently, 5 = always.

Student survey responses indicated that the instructors assigned to the treatment group were more likely to spend class time going over homework problems than the instructors assigned to the control group (see Table 14). Instructor surveys indicated that control group instructors spent slightly more time on homework than treatment group instructors during the first half of the semester, with the reverse occurring during the second half of the semester (see Table 15).

Table 14

Responses to Student Survey Items Concerning Class Coverage of Homework

Mid-Semester survey items	**Treatment mean**	**Control mean**	**df**	**t**
My instructor spends class time going over homework problems.	3.67	3.24	316	3.79***
End of semester survey item	**Treatment mean**	**Control mean**	**df**	**t**
My instructor spent class time going over homework problems.	3.59	2.93	322	5.29***

Note. the scale used for these survey items was: 1 = never, 2 = rarely, 3 = sometimes, 4 = frequently, 5 = always.
***$p < .001$

Table 15

Responses to Instructor Survey Items Concerning Class Coverage of Homework

Mid-Semester survey item	Treatment mean	Treatment SD	Control mean	Control SD
How much class time do you spend going over homework?	7.5 min.	2.5 min.	9.2 min.	5.2 min.
End of semester survey item	**Treatment mean**	**Treatment SD**	**Control mean**	**Control SD**
How much class time do you spend going over homework?	7.5 min.	2.5 min.	6.3 min.	2.5 min.

Classroom observation data indicated that the amount of time spent on homework was approximately equal in both groups (see Table 16).

Table 16

Observation Data for Class Coverage of Homework

Group	Number of observations	Range	Mean	Standard deviation
Treatment	6	5-22 min.	7.5 min.	7.5 min.
Control	9	5-17 min.	8.0 min.	7.9 min.

Instructor surveys and classroom observations also served to confirm the administration of the treatment. The question, "I assign homework according to the syllabus" appeared on both the mid-semester and end of semester survey. The mean response on the mid-semester survey was 5 (strongly agree). The mean response on the end of semester survey was 4.7 (4 = agree and 5 = strongly agree). In addition, classroom observations confirmed that instructors were following the syllabus for their particular group. During all observations, homework was distributed and collected and students appeared to be familiar with the homework collection and distribution routines set up by the individual instructors.

Analysis of study time data indicated that, up until the fourth block of lessons (i.e., after the third exam), both groups spent approximately equal amounts of time studying and completing homework (see Table 17). Time spent studying for exams was approximately equal for both groups (see Table 18).

Table 17

Analysis of Average Study Times for Lesson Blocks

Lesson block	Treatment mean	Control mean	df	t
Block I: Lessons 1-8	61.5 min.	62.6 min.	342	0.28
Block II: Lessons 9-19	49.8 min.	52.5 min.	320	1.18
Block III: Lessons 20-29	60.3 min.	60.1 min.	305	0.08
Block IV: Lessons 30-42	58.2 min.	49.9 min.	274	2.88**

**$p < .01$

Table 18

Analysis of Study Times for Exams

Exam	Treatment mean	Control mean	df	t
1st Exam	88.4 min.	84.5 min.	333	0.59
2nd Exam	95.4 min.	97.4 min.	296	0.23
3rd Exam	117.6 min.	116.9 min.	305	0.08
4th Exam	100.8 min.	93.2 min.	274	0.77
Final Exam	198.1 min.	235.9 min.	128	1.30

All p values > .20.

An analysis of variance via multiple regression (see Cohen & Cohen, 1983) revealed that students studied significantly longer for the third exam than for any of the other hourly exams, $F = 9.84$, $df = 3, 1212$, $p = 0$. Table 19 shows the pair-wise comparisons between average study times for the hourly exams.

Table 19

Pair-Wise Analysis of Study Times for Hourly Exams

Exams being compared	df	t
first and second	1212	1.73
first and third	1212	5.33**
first and fourth	1212	1.81
second and third	1212	3.25**
second and fourth	1212	0.11
third and fourth	1212	3.30**

Note. Pair-wise t-tests accomplished as described by Cohen and Cohen (1983).
**$p < .01$

Other Analyses

Quality of Homework: Treatment versus Control

Analysis of homework scores indicated that, until the middle of the semester, the treatment group received significantly better homework scores than the control group. The data reveal that the two groups had approximately equal homework scores during the second half of the semester. When homework scores were averaged over the entire semester, the difference between treatment and control group scores was not statistically significant (see Table 20).

Table 20

Mean Homework Scores

Block	Number of assignments	Treatment mean	Control mean	df	t
I	7	1.51	1.36	349	5.51***
II	7	1.62	1.54	339	1.97*
III	7	1.47	1.48	336	0.15
IV	9	1.35	1.43	331	1.41
Total	30	1.48	1.45	331	0.88

Note. The scale used to score homework was: 0 = less than one-third complete and correct, 1 = one-third to two-thirds complete and correct, and 2 = more than two-thirds complete and correct.
*$p < .05$ ***$p < .001$

Quality of Treatment Group Homework: New versus Review Topics

Each treatment group homework assignment received three scores: (a) the total score; (b) a score for the review problems; and (c) a score on current topic problems. Statistical t-tests revealed that, until the middle of the semester, the treatment group received significantly better scores on new topic homework problems than review topic homework problems. During the second half of the semester, the trend changed. There was no significant difference in new versus review homework scores during the third block. Scores on review problems were significantly better than scores on new problems during the fourth block. When homework scores were averaged over the entire semester, scores on new homework topics were significantly better than scores on review homework topics (see Table 21).

Table 21

Mean Homework Scores: New Topic versus Review Topics

Block	Number of assignments	Mean for new topic	Mean for review topic(s)	*df*	*t*
I	6	1.78	1.49	160	13.40***
II	7	1.73	1.53	160	7.54***
III	7	1.58	1.59	160	0.47
IV	9	1.46	1.58	153	4.89***
Total	29	1.65	1.56	153	7.37***

Note. The scale used to score homework was: 0 = *less than one-third complete and correct*, 1 = *one-third to two-thirds complete and correct*, and 2 = *more than two-thirds complete and correct*.
$^{***}p < .001$

Homework Error Analysis: Treatment versus Control

A sample of two control group sections and two treatment group sections was selected for a detailed error analysis on all of the homework problems in three key topic areas. The purpose of this analysis was to determine whether the distributed practice treatment enhanced the ability of the treatment group to correct their misconceptions (Ausubel, 1966). For the treatment group, 40% of the homework on a given topic was assigned on the day the topic was introduced. Because homework was collected and graded, students in the treatment group received feedback on the first 40% of the homework problems before attempting the remainder of the homework on that topic. Table 22 shows that students in the treatment group made fewer repeat errors on all error types except for simplification errors.

Table 22

Analysis of Repeat Errors

Group	Treatment group			Control group		
Error type	Number of students making the error on first 40% of problems	Number of students making the error on last 60% of problems	Repeat error rate	Number of students making the error on first 40% of problems	Number of students making the error on last 60% of problems	Repeat error rate
Improper simplification	11	9	81.8%	13	9	69.2%
Improper substitution	10	4	40.0%	6	3	50.0%
Concept error	5	2	40.0%	0	0	
Sign error	12	2	16.7%	5	2	40.0%
Failed to simplify	1	0	0.0%	2	1	50.0%
Total	39	17	43.6%	26	15	57.7%

Effect of Homework on Exam Scores

Five separate regressions were performed to determine whether homework scores could predict a significant proportion of variance in exam scores. Block homework scores explained a statistically significant proportion of variance in all hourly exam scores. Similarly, the total homework score explained a statistically significant proportion of variance in the final exam score (see Table 23).

Table 23

Effect of Homework on Exam Scores

Exam	r	R^2	df	F
1st Exam	.39	.151	1, 349	62.07***
2nd Exam	.33	.109	1, 339	41.54***
3rd Exam	.33	.109	1, 336	41.22***
4th Exam	.30	.090	1, 331	32.67***
Final Exam	.39	.153	1, 315	56.96***

***$p < .001$

CHAPTER 5

DISCUSSION AND CONCLUSIONS

Principal Findings and Discussion

Distributed Practice Main Effect

While this study hypothesized that there would not be a significant main effect due to the treatment, the reverse was shown. The distributed practice treatment produced a statistically significant main effect on four out of six measures of Precalculus achievement (three hourly exams and the final course percentage grade). These findings are in agreement with results reported by Friesen (1975), Parker (1990), Peterson (1970), Reed (1983; Klingele & Reed, 1984), and Saxon (1982). The treatment did not produce a statistically significant main effect on the third exam or final exam.

Effect sizes were calculated to better interpret the practical significance of the distributed practice treatment. Effect sizes were found to be smaller than predicted. The treatment produced an effect size (f^2) of 0.013 on the first exam, 0.029 on the second exam, 0.035 on the fourth exam, and 0.040 on the final course percentage grade.

Because the effect sizes were smaller than hypothesized, ($f^2 = .11$), the statistical tests were not as powerful as predicted. The power of the tests of the main treatment effect ranged from approximately 30% (first exam) to approximately 70% (final course percentage grade).

Still, the differences in test scores between the treatment and control groups favored the treatment group in every case. A mean difference of 5.13 percentage points on the first, second, and fourth exam translates to an advantage of about a half of a letter grade for students in the treatment group. In addition, smaller standard deviations for the treatment group may indicate that the distributed practice treatment served to eliminate the extremely low scores (refer to Table 6 in Chapter 4).

Because the distributed practice treatment produced a significant main effect on all but one of the hourly exams, a plausible explanation for this aberration was sought. The experimental and control groups achieved nearly equal scores on the third exam (treatment mean = 70.71 and control mean = 70.27). Although the two groups spent nearly equal time studying for the exam (treatment mean = 117.6 minutes and control mean = 116.9 minutes), both groups reported spending significantly more time studying for the third exam than they spent studying for any of the other three hourly exams. The third exam occurred after mid-semester progress reports which may have motivated students to devote more time to studying for the exam. The additional study time on the third exam allowed the control group to match the achievement of the treatment group. It is possible that the additional study time imitated the distributed practice treatment by allowing for more repetitions of problem sets.

Oddly, the distributed practice treatment did not have a significant effect on final exam scores. An explanation for this deviation was also sought. One possible cause for the disparity was the exemption of the top 16 students from the final exam. Eleven of the exempted students were from the treatment group with only five from the control group. It is likely that the treatment group would have outscored the control group on the final exam if these top performers had taken the exam. Study time for the final exam may have also influenced the results. When compared to study times for hourly exams, study times for the final exam were much longer. In addition, the final exam was scheduled late during final exam week making study time for the exam not only longer, but more widely distributed. The benefits of the longer and more dispersed study time may have been similar to the benefits experienced by the treatment group.

Aptitude-Treatment Interaction Effects

Distributed practice homework assignments benefited the students in the treatment group without regard for prior mathematics achievement or mathematics anxiety. ATI theory suggests that the treatment might be detrimental to high-ability students. This study and a previous study by the author (Revak, 1994) suggest that the distributed practice treatment is beneficial to low- and average-track students. Gehlbach (1979) advocated AT correction - instructional treatments designed to eliminate ATIs. Because the distributed practice treatment was effective for students with low and high prior achievement and low and high mathematics anxiety, this study achieved ATI elimination (Gehlbach, 1979; Snow, 1992).

Two significant two-way interactions were expected: (a) Prior Mathematics Achievement × Treatment, and (b) Mathematics Anxiety × Treatment. Neither of these interactions was found to explain a significant proportion of variance in Precalculus achievement above what had already been explained by the covariates and the distributed practice treatment.

The sample in this study, first year students on the low mathematics ability track at the Air Force Academy, may provide some explanation for the lack of significant interaction effects. Students on the average track are typically enrolled in Calculus I during the Fall semester and Calculus II during the Spring semester. Similarly, those with high math ability are usually enrolled in Calculus II or Calculus III during the Fall semester. Because mathematics achievement has been found to correlate negatively with mathematics anxiety (Adams & Holcomb, 1986; Berenson et al., 1992; Clute, 1984; Coleman, 1991; Cooper & Robinson, 1989; Covington & Omelich, 1987; Frary & Ling, 1983; Gliner, 1987; Hembree, 1990; Lawson, 1993; McCoy, 1992; Richardson & Suinn, 1972), the students placed into Precalculus were probably relatively high in mathematics anxiety. Aptitude-treatment interactions are not expected to be as strong when students have comparable aptitudes. The homogeneity of this group may have nullified the expected two-way interaction effects.

The results of this study challenge the results reported by Hirsch and his colleagues (1982, 1983). Hirsch et al. found significant Prior Achievement × Treatment ATIs on three out of five measures of Calculus I achievement. In all three cases, the distributed practice treatment was beneficial to students scoring at or below the mean on an algebra and analytic geometry pre-test. It is not known whether the students in Hirsch's study were grouped homogeneously.

Additional Findings and Discussion

Students in the treatment group made fewer repeat errors than students in the control group. This study supports Ausubel's (1966) contention that distributed practice allows students to clarify ambiguities and correct misconceptions. Although the treatment group committed fewer repeat errors than the control group, the rate of repeat errors is still large enough to cause concern (16.7% to 81.8%). The high rate of repeat errors suggests that students may not have reaped the full value of the feedback provided on the graded homework.

Students in the treatment group achieved higher scores on homework problems based on new topics than on homework problems based on review topics. Because the homework problems were assigned sequentially from the textbook, the problems associated with the most recent topics were easier than the problems associated with the review topics. In addition, students may have been more comfortable with traditional (vertical) homework assignments and may have attempted the problems associated with the new topic first. Review topic homework problems may have been relegated to the back burner or attempted when students were fatigued or frustrated.

During the last block of classes, students in the treatment group reported spending more time studying than students in the control group. This is most likely due to the increasing length and difficulty of the treatment group's homework assignments as the semester progressed.

Limitations

This study was limited by the length of the semester and the number of homework assignments. By following the homework pattern advocated by Hirsch and his colleagues (1982, 1983), homework for topics introduced after the tenth lesson could not be fully distributed. Homework for each topic was assigned in the order listed in the textbook, with the easier problems preceding the more difficult ones. For the treatment group, this meant that the easiest problems were assigned early in the distribution pattern with the hardest problems assigned in the later stages of the distribution. The treatment may have been more effective if the difficulty level of problems within each assignment was mixed. Similarly, the distributed practice treatment may be more effective when applied to courses of longer duration.

Several factors may limit the internal validity and generalizability of this study. Although the sample was large, the subjects, being military academy cadets, may not be representative of typical high school or college students. Overall, students attending the USAFA are a fairly homogeneous group with similar academic and career goals.

The limited external validity due to the controlled atmosphere at the Air Force Academy serves to strengthen the internal validity of the study. First year USAFA cadets have little opportunity to leave campus. Cadets are issued uniforms, textbooks, and a personal computer. All cadets live in the dormitories, eat at the dining hall, and are subject to rigorous military and athletic training in addition to academic course work. USAFA cadets are all between the ages of 17 and 21. Cadets are not permitted to be married, have children, or hold part-time jobs. All first year cadets are required to take the same core courses and carry similar credit hour loads. Cadets are not permitted to skip classes, and exams are missed only in extreme circumstances. Threats due to subject characteristics, mortality, location, history, and subject attitude have been minimized due to the conditions stated above (Fraenkel & Wallen, 1993).

Certain threats to internal validity remain. Although it cannot be assumed that instructors with similar experience levels are equally effective, this study and a previous study conducted at the USAFA found that instructor experience level was not a significant contributor to achievement variance (Thompson et al., 1979). It is possible that one or more instructors were biased, either for or against the distributed practice treatment. Furthermore, some of the differences in achievement may have been due to implementer characteristics and not the treatment (Fraenkel & Wallen, 1993). In addition, the mid-semester change of instructor may have positively or negatively impacted four of the control group sections. A Hawthorne effect may have resulted if the students in the treatment group recognized that they were receiving special treatment in the way of distributed practice homework assignments (Fraenkel & Wallen). Conversely, students assigned to the control group may have suffered a demoralization effect (Fraenkel & Wallen). In addition, the treatment may have had a negative impact on the achievement of the experimental group if exam items were related to homework problems not yet assigned due to the distributed practice treatment. Finally, it is possible that the treatment was not fully confined to the treatment group. Students from different sections shared dormitory rooms and may have studied together. Although it is possible that students discussed homework problems, survey responses indicated that students rarely studied with students who used a different syllabus.

The USAFA policy of exempting top performers from the final exam rendered the final exam data incomplete. Of the 16 exempted students, 11 were from treatment group sections and 5 were from control group sections. This absence of data may have biased the results in favor of the control group.

Students in both groups were expected to complete exploratory exercises (margin exercises) prior to each class. Although these exercises were not collected or graded, many of the instructors did a cursory check of margin exercises before starting class. The addition of margin exercises to the daily homework assignments may have modified the descriptions of the experimental and control treatments. Instead of examining and comparing the effects of the vertical (traditional) and distributed homework patterns, this research may have studied the effects of the oblique and semi-oblique homework patterns (refer to Figure 1 in Chapter 1). If students dutifully completed the margin exercises in addition to assigned homework, better achievement for both groups should have resulted.

The attempt to measure an affective construct like mathematics anxiety could also threaten the internal validity of the study. Jones, Sensening, and Haley (1974) argued that "the nature of the self-descriptions obtained is in part a function of the setting or situation in which they were obtained" (p. 44). In addition, students who feared that they would be judged negatively or criticized may have been less than frank when responding to the MARS survey items (Cope, 1988; Henerson, Morris, & Fitz-Gibbon, 1987). There is also the possibility that some students misunderstood MARS items or that students were not fully aware of their feelings about mathematics anxiety (Bessant, 1995). Finally, mathematics anxiety as measured by the survey may have had only a weak link to math anxious behavior (Dew et al., 1983; Henerson et al., 1987; McLeod, 1992). Still, Henerson et al. (1987) recommend the use of self-report attitude surveys "unless you have reason to believe that the people whose attitudes you are investigating are unwilling or unable to provide the necessary information" (p. 20).

Recommendations for Future Research

Distributed practice homework has been shown to be beneficial to students on the low or average mathematics track at the USAFA. Testing of the distributed practice treatment on high ability students is recommended. In addition, different variations of spaced review should be investigated across a wide variety of students, institutions, and mathematics courses. Because the collection and grading of homework may have caused a higher than average homework completion rate, this study should be replicated in an environment where homework is not collected.

Future studies of this kind should include the study time variable. The study time data in this study indicate that the distributed practice treatment had the greatest impact when less time was devoted to studying for an exam. An analysis of how students use their study time could help shed light on these findings. Because the distributed practice treatment was effective for improving the performance of college students, its effectiveness should be investigated in middle school and high school settings where testing is more frequent and study times generally shorter.

This research has suggested that the distributed practice treatment decreases the number of repeat errors. This portion of the study should be replicated with a larger sample and a more rigorous research design.

According to Holtan (1982), the value of the distributed practice treatment may well be in the delayed retention of the skills and concepts practiced. Follow-up retention tests are recommended for the students taking part in this study.

The multiple correlations revealed in this study accounted for less than 26% of the variance in all measures of achievement. This suggests that the contribution of other variables such as motivation, attitude, and study habits should be examined. Systematic research in this area should help identify the students who will benefit most from distributed practice assignments and contribute to the theoretical structure of ATI.

Summary

This study has documented a significant positive correlation between homework scores and exam scores. Homework scores were found to account for between 10% and 15% of the variability in exam scores. Meaningful homework may be viewed as an important component in mastering mathematics course material.

The distributed practice treatment has shown to be beneficial to students in the low and average ability mathematics tracks at the Air Force Academy. The Department of Mathematical Sciences at the Air Force Academy believes that the spaced review will benefit most students and has mandated distributed practice homework assignments for all core Calculus courses (Precalculus, Calculus I, and Calculus II).

Enrollments in remedial mathematics college courses are on the rise (Berenson et al., 1992) and 90% of college mathematics enrollments are in elementary calculus, elementary statistics, and courses prerequisite to them (National Research Council, 1989). There is great potential for application of the distributed practice model. Mathematics achievement is still the principal gateway for students preparing to enter technical and scientific careers, and distributed practice may help foster success in these pivotal math courses.

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APPENDIX A

ACADEMIC TESTING MATERIAL
Math 130 -- Graded Review I

Name ____________________________ Instructor____________________

Section ______ Start Time ________ Stop Time ________

Study Time (mins) ________

Points: 675 Time: 50 minutes

Directions

DO NOT Open Exam Booklet Until Instructed To Do So!

1. Write your name, instructor's name, section, and study time on this sheet.

2. Ensure you have your own digitek and that your social security number is coded.

3. Calculators may be used, but <u>not</u> shared.

4. You have 50 minutes to complete this exam. Start and stop times will be written on the board. You must remain in this room until the end of the exam period.

5. Your digitek **<u>must</u>** be completely coded when "cease work" is called. Failure to accurately complete your digitek will result in a 30 point penalty (1 MC question).

	PROBLEM	POINTS	SCORE
Multiple Choice	1-10	300	______
Workout	11	65	______
	12	55	______
	13	65	______
	14	60	______
	15	65	______
	16	65	______
TOTAL		675	______

Multiple Choice: Exam items 1-10 are multiple choice. Circle the **one** most correct answer for each item on your test booklet AND digitek under **TEST A.** Each item is worth 30 points.

(30) 1. Simplify $\dfrac{3x^2y^5z^{-7}}{15y^{-3}z^{12}}$

a. $5x^2y^2z^5$

b. $5x^2y^8z^{19}$

c. $\dfrac{1}{5}\left(\dfrac{x^2y^2}{z^{19}}\right)$

d. $\dfrac{y^2}{5z^{19}}$

e. $\dfrac{x^2y^8}{5z^{19}}$

(30) 2. Solve $\dfrac{1}{t}+\dfrac{1}{2t}+\dfrac{1}{5t}=34$ for t

a. $\dfrac{1}{272}$

b. $\dfrac{4}{85}$

c. $\dfrac{1}{20}$

d. 20

e. $\dfrac{85}{4}$

(30) 3. Determine the meaningful replacements for x in the expression $\sqrt{8x-3}$

a. $x \geq 0$

b. $x \geq -\frac{3}{8}$

c. $x \geq \frac{3}{8}$

d. $x \leq -\frac{3}{8}$

e. $x \leq \frac{3}{8}$

(30) 4. Multiply and simplify $-2(c^2 - c + 2)(c - 4) =$

a. $-2c^3 - 10c^2 - 12c + 16$

b. $-2c^3 - 10c^2 + 12c - 16$

c. $-2c^3 + 10c^2 + 12c + 16$

d. $-2c^3 + 10c^2 - 12c + 16$

e. $-2c^3 - 6c^2 + 6c + 16$

(30) 5. The least common denominator needed to express $\frac{2}{3x}+\frac{x}{x+2}-\frac{16x-3}{x^2-4}$ as a single fraction is:

a. $(x+2)(x^2-4)$

b. $3(x+2)(x^2-4)$

c. $3x(x+2)(x^2-4)$

d. $3(x+2)(x-2)$

e. $3x(x+2)(x-2)$

(30) 6. Simplify $\left|-3wx^3\right|$ by evaluating or removing the unneeded absolute value signs.

a. $3w|x^3|$

b. $3x^3|w|$

c. $3x^2|w||x|$

d. $3wx^3$

e. $-3wx^3$

(30) 7. Factor $y^3 + 64$ until it cannot be factored further.

a. $(y+4)^3$

b. $(y+4)(y^2 - 4y + 16)$

c. $(y-4)(y^2 + 4y + 16)$

d. $(y+4)(y^2 - 4y - 16)$

e. It cannot be factored further.

(30) 8. What are all values of x for which the inequality $5x + \frac{5}{3} \leq -2x - \frac{2}{3}$ is true?

a. $x \geq 0$

b. $x \geq -\frac{1}{3}$

c. $x \leq \frac{1}{3}$

d. $x \leq -\frac{1}{3}$

e. $x \leq -\frac{7}{9}$

(30) 9. A quadratic equation with solutions $x = \frac{3}{2}$ and $x = -5$ is:

a. $2x^2 + 7x - 15 = 0$

b. $2x^2 - 7x - 15 = 0$

c. $2x^2 + 13x + 15 = 0$

d. $x^2 + 7x - 30 = 0$

e. $x^2 - 7x - 30 = 0$

(30) 10. Which of the following equations are always true?

I) $\sqrt{a^2 - b^2} = a - b$

II) $\frac{a+b}{b} = a$

III) $\frac{a+b}{c+d} = \frac{a}{c} + \frac{b}{d}$

a. None are true

b. I, II are true

c. I, III are true

d. II, III are true

e. All (I, II, III) are true

Workout: Questions 11 - 16 are workout items. Partial credit will be awarded for work that is appropriate to the item. Therefore, show your work!

(65) 11. Perform the indicated operations and simplify the result.

$$\frac{\left(\dfrac{x^2-14x+45}{x^2-49}\right)}{\left(\dfrac{x-9}{x^2-7x}\right)}$$

(55) 12. Factor $cx^2+cy+dx^2+dy$ until it cannot be factored further.

(65) 13. In the equation $x^4+15x^2-16=0$, find all real values of x which satisfy the equation. Express the solution in set builder notation.

(30) 14a. Complete the following table for the discriminant:

Discriminant	Describe the Nature & Number of Solutions
$b^2 - 4ac = 0$	______________________
$b^2 - 4ac > 0$	______________________
$b^2 - 4ac < 0$	______________________

(30) 14b. Explain the benefits of evaluating the discriminant of a quadratic equation before attempting to solve. **CAUTION:** You will be graded on clarity as well as completeness of your answer. Be sure your entire answer fits in the provided box. Anything outside the box will not be graded.

Graded Answer:____

(65) 15. In your Math 130 class, you have scores of 70%, 82%, 85% , and 89% on the first four of five quizzes. To get a grade of B, the average on the five quizzes must be greater than or equal to 80% and less than 90%. Solve using the Problem Solving Approach from class to find the range of scores you will need on the last quiz to get a B.

Put the answer into a complete sentence or sentences using the context of the problem. (i.e. what does your range mean in terms of the required quiz score).

Sentences:____

(65) 16. Rewrite and simplify as much as possible the expression $\sqrt[5]{(x+y)^4} \cdot \sqrt{x+y}$ using a single radical.

ACADEMIC TESTING MATERIAL
Math 130 - Graded Review II

Name ____________________ Instructor ____________________

Section ______ Start Time ________ Stop Time ________

Study Time (mins) ________

Points: 675 Time: 50 minutes

Directions

DO NOT Open Exam Booklet Until Instructed To Do So!

1. Write your name, instructor's name, section, and study time on this sheet.

2. Ensure you have your own digitek and that your social security number is coded.

3. Calculators may be used, but not shared.

4. You have 50 minutes to complete this exam. Start and stop times will be written on the board. You must remain in this room until the end of the exam period.

5. Your digitek **must** be completely coded when "cease work" is called. Failure to accurately complete your digitek will result in a 35 point penalty (1 MC question).

	PROBLEM	POINTS	SCORE
Multiple Choice	51-58	280	______
Workout	59	80	______
	60	80	______
	61	80	______
	62	80	______
	63	80	______
TOTAL		675	______

Multiple Choice: Exam items 51-58 are multiple choice. Circle the **one** most correct answer for each item on your test booklet AND digitek under **TEST B.** Each item is worth 35 points.

(35) 51. If $f(x) = 3x - 4$ and $g(x) = x^2 - x$, what is $f(g(x))$?

a. $9x^2 - 27x + 20$

b. $x^2 + 2x^4$

c. $3x^2 - x - 4$

d. $3x^2 - 3x$

e. $3x^2 - 3x - 4$

(35) 52. If $f(x) = \dfrac{x+3}{5-x}$, then $f(a+4) =$

a. $\dfrac{a+7}{1-a}$

b. $-\dfrac{a+7}{9-a}$

c. $\dfrac{38-6a}{5-a}$

d. $-\dfrac{23-3a}{5-a}$

e. $\dfrac{a+3}{5-a}$

(35) 53. The domain of $f(x)=\dfrac{1}{\sqrt{3x+8}}$ is:

a. $\{x|x>0\}$

b. $\left\{x\middle|x>-\dfrac{8}{3}\right\}$

c. $\left\{x\middle|x\geq-\dfrac{8}{3}\right\}$

d. $\left\{x\middle|x<-\dfrac{3}{8}\right\}$

e. $\{x|x\in R\}$

(35) 54. Solve for x: $2x-9\sqrt{x}+4=0$

a. The x solutions are complex

b. $\dfrac{1}{2}$, 4

c. $\dfrac{5}{2}$, 8

d. $\dfrac{1}{4}$, 16

e. $\dfrac{25}{4}$, 64

(35) 55. A point-slope form of the equation of the line through (2, -3) and **perpendicular** to the line $4x-3y=6$ is:

a. $y+3=-\frac{3}{4}(x-2)$

b. $y-3=-\frac{3}{4}(x+2)$

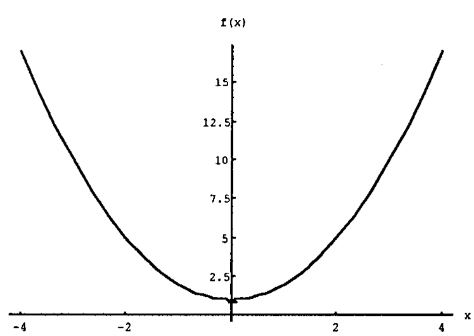
c. $y+3=-\frac{4}{3}(x-2)$

d. $y+3=\frac{4}{3}(x-2)$

e. $y-3=\frac{4}{3}(x+2)$

Questions 56 - 58 ask you to compare two quantities, one in **COLUMN A** and one in **COLUMN B**. Determine whether

a. the quantity in **COLUMN A** is **GREATER** than the quantity in **COLUMN B**.
b. the quantity in **COLUMN B** is **GREATER** than the quantity in **COLUMN A**.
c. the two quantities are **EQUAL**.
d. the relationship cannot be determined from the information given.

GIVEN	Column A	Column B
(35) 56. $h(y) = y^3 - y^2 - y$	$h(-1)$	$h(1)$
(35) 57. $2x + 13 = 10$	$4x + 26$	20
(35) 58. Given the graph of $f(x)$: 	$f(x+3)$	$f(x-3)$

Workout: Questions 59 - 63 are workout items. Partial credit will be awarded for work that is appropriate to the item. Therefore, show your work.

(80) 59. Solve the following equation for *x* and put your answer in set builder notation.

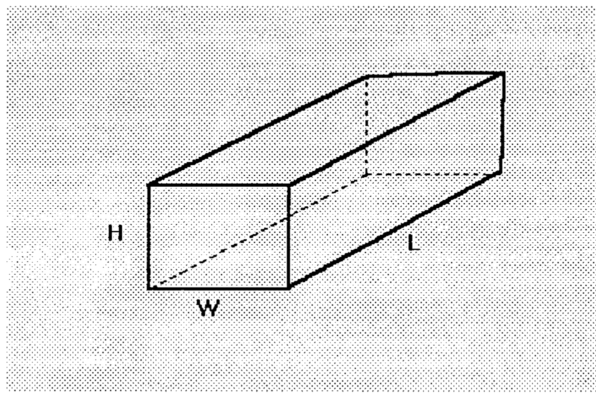
$$\frac{x^2-2}{x-1}=x+1-\left(\frac{1}{x-1}\right)$$

(80) 60. If $f(x)=2x-5x^2$, find $\dfrac{f(x+h)-f(x)}{h}$. Simplify your answer as much as possible.

(75) 61. How many *x*-intercepts can the graph of a function have? How many *y*-intercepts can it have? Justify your answers.

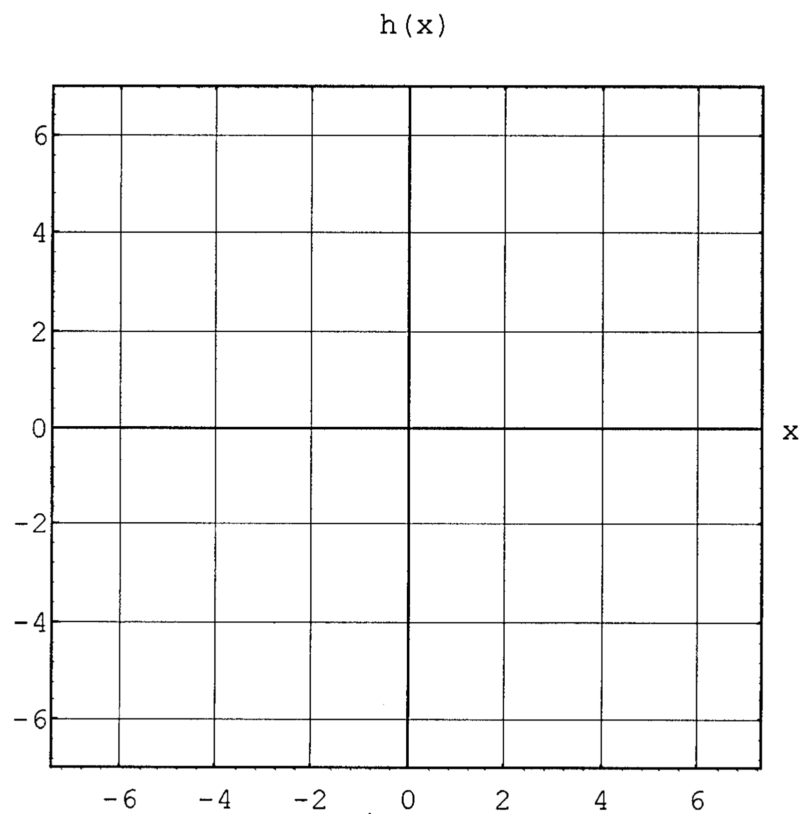
GRADED ANSWER:

(80) 62. The safe load of the horizontal beam, shown below, supported at both ends varies jointly as the width and the square of the height, and inversely as the length of the beam. If a 2x4 in. beam, 8 ft long will support 500 lbs safely, what is the safe load for a 4x8 in. beam of the same material 20 ft long (Include units)?



(80) 63. Graph the following function:

$$h(x) = \begin{cases} -x^2 + 1 & \text{for} \quad x < -1 \\ 2x + 3 & \text{for} \quad -1 \leq x < 1 \\ \sqrt{x+3} & \text{for} \quad x > 1 \end{cases}$$



ACADEMIC TESTING MATERIAL
Math 130 - Graded Review III

Name ____________________ Instructor ____________________

Section ______ Start Time ________ Stop Time ________

Study Time (mins) ________

Points: 675 Time: 50 minutes

Directions

Do Not Open Exam Booklet Until Instructed To Do So!

1. Write your name, instructor's name, section, and study time on this sheet.

2. Ensure you have your own digitek and that your social security number is coded.

3. Calculators may be used, but <u>not</u> shared.

4. You have 50 minutes to complete this exam. Start and stop times will be written on the board. You must remain in this room until the end of the exam period.

5. Your digitek <u>**must**</u> be completely coded when "cease work" is called. Failure to accurately complete your digitek will result in a 40 point penalty (1 MC question).

	PROBLEM	POINTS	SCORE
Multiple Choice	101 - 108	320	______
Workout	109	75	______
	110	75	______
	111	65	______
	112	65	______
	113	75	______
TOTAL		675	______

Multiple Choice: Exam items 101-108 are multiple choice. Circle the **one** most correct answer for each item on your test booklet AND digitek under **TEST C.** Each item is worth 30 points.

(40) 101. Given the quadratic function $g(x) = 2x^2 - 4x + 5$, locate the vertex and decide whether it is a maximum or minimum.

a. Vertex: (-1, 11), minimum

b. Vertex: (1, 3), minimum

c. Vertex: (1, 3), maximum

d. Vertex: (1, 4), minimum

e. Vertex: (1, 4), maximum

(40) 102. Some values of functions f and g are given in the table below. The value of $g(f(3))$ is:

a. 1

b. 2

c. 3

d. 4

e. insufficient information is given

x	f(x)	g(x)
1	1	3
2	3	4
3	4	2
4	2	1

(40) 103. Which of the following statements is **NOT** true for rational functions?

a. Rational functions are continuous everywhere except where the denominator is undefined (equal to zero).

b. Horizontal and oblique asymptotes are always linear functions.

c. A rational function cannot have both horizontal and oblique asymptotes.

d. A function can never cross a vertical asymptote, however, it may cross horizontal and oblique asymptotes.

e. Horizontal asymptotes occur where the denominator is zero.

(40) 104. Which of the following can be the equation of the graph shown below?

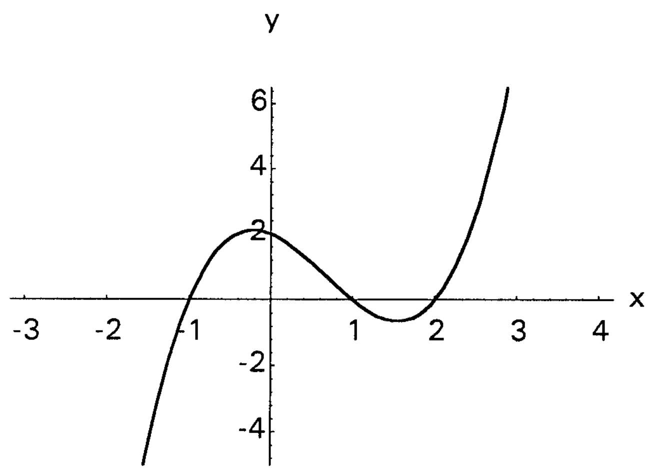
a. $y = (x-1)(x-2)$

b. $y = (x+1)(x+2)$

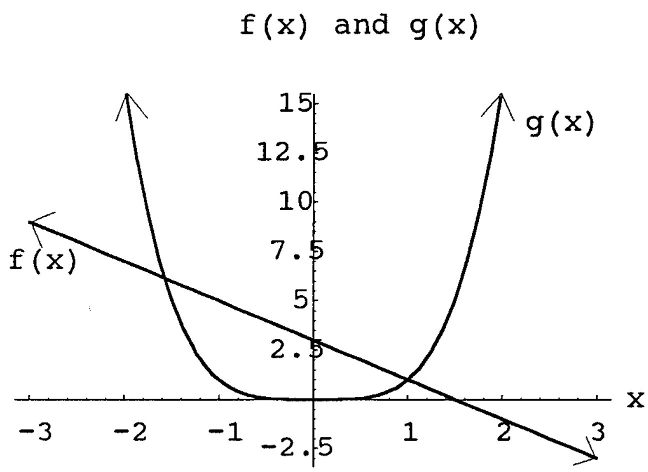
c. $y = (x-1)(x+1)(x-2)$

d. $y = (x-1)(x+1)(x+2)$

e. $y = (x-1)(x-1)(x+1)(x+2)$



(40) 105. The graphs of two functions, $f(x)$ and $g(x)$, are shown below. For which values of x is $f(x)$ less than or equal to $g(x)$?



a. $[1, \infty)$

b. $[-1.5, 1]$

c. $(-1.5, 1)$

d. $(-\infty, -1.5] \cup [1, \infty)$

e. $(-\infty, -1.5] \cap [1, \infty)$

(40) 106. Find a polynomial of lowest degree having roots 0 and 5, and having -3 as a root of multiplicity 2.

a. $f(x) = (x+2)(x+3)(x-5)$

b. $f(x) = x(x+2)(x+3)(x-5)$

c. $f(x) = x(x-3)(x-3)(x+5)$

d. $f(x) = x(x+3)(x+3)(x-5)$

e. $f(x) = x(x+2)(x+3)(x+3)(x-5)$

(30) 107. Solve for x: $|3x+2| \le 5$

a. $-1 \le x \le 1$

b. $-1 \le x \le \frac{7}{3}$

c. $-\frac{7}{3} \le x \le 1$

d. $x \le -\frac{7}{3}$ or $x \ge 1$

e. $x \le -1$ or $x \ge \frac{7}{3}$

(40) 108. The graph of the function $y = \dfrac{2x-3}{x+6}$ has the following characteristics:

a. Vertical Asymptote: $x = \dfrac{3}{2}$; Horizontal Asymptote: none

b. Vertical Asymptote: $x = \dfrac{3}{2}$; Horizontal Asymptote: x axis

c. Vertical Asymptote: $x = -6$; Horizontal Asymptote: none

d. Vertical Asymptote: $x = -6$; Horizontal Asymptote: x axis

e. Vertical Asymptote: $x = -6$; Horizontal Asymptote: $y = 2$

Workout: Questions 109 - 113 are workout items. Partial credit will be awarded for work that is appropriate to the item. Therefore, show your work.

(75) 109. The sum of the base and the height of a triangle is 20 cm. Find the dimensions for which the area is a maximum.

(75) 110. Use interval notation to specify where:

$$\frac{2x-1}{x+7} \geq 1$$

(65) 111. Explain the differences between a polynomial function and a rational function with regards to domain and asymptotes. **ONLY WORK IN THE BOX WILL BE GRADED!**

Graded Answer: ______________________________

(65) 112. Sketch a function $f(x)$ such that it meets all the following requirements:

x-intercept @ $-\frac{1}{2}$

y-intercept @ -4

Real Roots @ -2, 3

Polynomial with 2 turning points

(75) 113. Find the value of *k* such that the line

$$(k-1)x+(k+1)y-7=0$$

is **perpendicular** to the line $3x+5y+7=0$

ACADEMIC TESTING MATERIAL
Math 130 - Graded Review IV

Name ____________________ Instructor __________________

Section _____ Start Time ______ Stop Time: ______

Study Time (mins) ________

Points: 500 Time: 50 minutes

Directions

Do Not Open Exam Booklet Until Instructed To Do So!

1. Write your name, instructor's name, section, and study time on this sheet.

2. SHOW ALL WORK TO RECEIVE FULL CREDIT!

3. You have 50 minutes to complete this exam. Start and stop times will be written on the board. You must remain in this room until the end of the exam period.

ITEM TYPE	Item Number	Points	Subscore
Multiple Choice	151 - 158	320	_______
	159	75	_______
	160	65	_______
	161	75	_______
	162	65	_______
	163	75	_______
TOTAL		675	_______

Multiple Choice: Exam items 151-158 are multiple choice. Circle the one correct answer for each item on your test booklet AND digitek under **TEST D.** Each item is worth 30 points.

(40) 151. Let $f(x) = 3x - 2$. What is $f(x+h) - f(x)$?

a. h

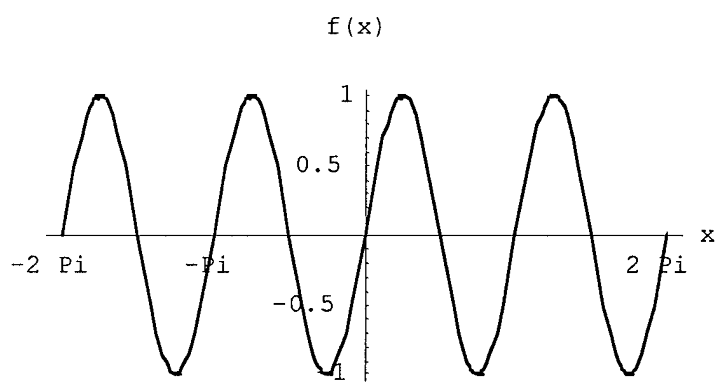
b. $3h$

c. $3h - 2$

d. $3h - 4$

e. $3x + 3h - 2$

(40) 152. Which of the choices below describes the alteration made to the graph of $f(x) = \sin x$, to produce the following graph:



a. The period of the graph was decreased to π units.

b. The period of the graph was increased to 4π units.

c. The graph was shifted π units to the left.

d. The graph was shifted up 1 unit.

e. The amplitude of the graph was doubled.

(40) 153. If (5, -12) is a point on the terminal side of θ, find csc θ:

a. $\frac{13}{12}$

b. $\frac{-13}{12}$

c. $\frac{12}{13}$

d. $\frac{-12}{13}$

e. $\frac{-5}{13}$

(40) 154. Simplify: $(\sin\theta)(\tan\theta)(\csc^2\theta)$:

a. $(\tan\theta)(\sin^2\theta)$

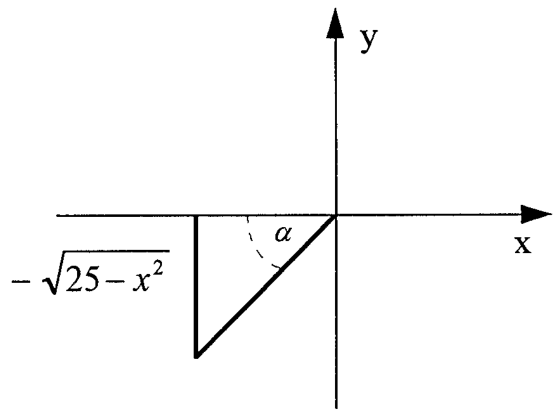
b. $\cos\theta$

c. $\sin\theta$

d. $\tan\theta$

e. $\sec\theta$

(40) 155. Use the following triangle to find tan α:



a. $\dfrac{x}{\sqrt{25-x^2}}$

b. $\dfrac{x}{-\sqrt{25-x^2}}$

c. $\dfrac{\sqrt{25-x^2}}{x}$

d. $\dfrac{-\sqrt{25-x^2}}{x}$

e. $\dfrac{5-x}{x}$

(40) 156. The zeros of the function $y = \dfrac{(2x+1)(x-1)}{x+1}$ are

a. -1

b. 1

c. $-\dfrac{1}{2}$ and 1

d. $\dfrac{1}{2}$ and -1

e. $-\dfrac{1}{2}$, 1, and -1

(40) 157. If $\sin\theta = -\cos\theta$ and if $\pi \le \theta \le 2\pi$ then $\theta =$

a. $\dfrac{2\pi}{3}$

b. $\dfrac{3\pi}{4}$

c. $\dfrac{5\pi}{6}$

d. $\dfrac{4\pi}{6}$

e. $\dfrac{7\pi}{4}$

(40) 158. Which of the following identities is NOT true?

a. $\cos(-x) = \cos x$

b. $\cos(x + 6\pi) = \cos x$

c. $\cos(\pi - x) = \cos x$

d. $\sin(x + \frac{\pi}{2}) = \cos x$

e. $\cos 2x = 2\cos^2 x - 1$

Workout: Questions 159 - 163 are workout items. Partial credit will be awarded to work that is appropriate to the item. **SHOW ALL WORK TO RECEIVE FULL CREDIT!**

(75) 159. A pitcher's earned run average, A, varies directly as the number of earned runs R allowed and inversely as the number of innings pitched I. In a recent year, a pitcher had an earned run average of 2.92. He gave up 85 earned runs in 262 innings. How many earned runs would he have given up had he pitched 300 innings (round to the nearest whole number)?

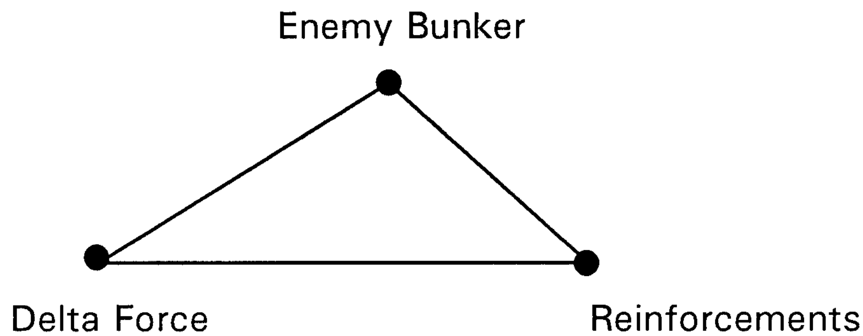
(65) 160. Prove $\left(\frac{\sin x}{1+\sin x}\right)\left(\frac{\cos x}{1-\sin x}\right) = \tan x$.

(75) 161. A pulley belt runs uncrossed around two pulleys of radii 12 in and 6 in respectively. If the larger pulley rotates at 16 radians/sec, what is the angular speed, IN REVOLUTIONS PER SECOND, of the smaller pulley?

(65) 162. Describe the effects of A, B, C, & D on $f(x) = A\cos(Cx - D) + B$. **ONLY WORK IN THE BOX WILL BE GRADED!**

GRADED WORK:

(75) 163. The Lieutenant (LT) of Delta Force, a special forces unit, observes an enemy bunker using night vision goggles. From earlier intel, the LT knows that the enemy bunker is at a bearing of 45° (clockwise from due north) and a range of $30\sqrt{2}$ km from their current position. Delta force also knows that the enemy's reinforcements are 90° (from due north) and (30+ $10\sqrt{3}$) km to the east of their position. The commander of the enemy bunker wants to know how far away his reinforcements are. USE EXACT VALUES FOR YOUR ANSWER (i.e. $\sqrt{3}$ NOT 1.7321).



(10) **BONUS:** What bearing from the reinforcements is the enemy bunker? (remember, all angles are measured clockwise from due north!)

APPENDIX B

Course Syllabus
Control Group

TEXT: Algebra and Trigonometry, by Keedy, Bittinger, & Beecher, Sixth Edition, 1993

Read each section and complete the margin exercises as instructed by the text. Do all margin exercises unless otherwise stated. Homework assignments will be completed using the designated format (see handout). All margin exercises, homework and other assignments are DUE ON the listed lesson.

BLOCK I - Basic Concepts of Algebra
Equations, Inequalities, and Problem Solving

Lesson	Date	Topic	ME Due	Other Assignments Due
1	10, 11 Aug	Intro/Admin Exponents	None 1.2	None
2	14, 15 Aug	Factoring Rational Expressions	1.5 (even) 1.6	pgs 18-19: 23, 26, 30, 43, 99, 104, 115,116
3	16, 17 Aug	Radical Notation Rational Exponents	1.7 (even) 1.8 (odd)	pgs 32-33: 9, 11, 13, 18 pgs 40-41: 15, 22, 34, 41, 47
4	18, 21 Aug	Solving Equations & Inequalities	2.1	pgs 49- 50: 7, 26, 41, 64, 75 pgs 55-56: 30, 36, 37, 54, 57
5	22, 23 Aug	Rational Equations	2.2	pgs 70-71: 7, 11, 14, 16, 17, 24, 45, 47, 50, 54
6	24, 25 Aug	Quadratic Equations Equations Reducible to Quadratic	2.5 2.8	pgs 76-77: 8, 15, 18, 19, 22, 25, 27,34
7	28, 29 Aug	Formulas for Problem Solving	2.3 2.6	pgs 104-105: 23, 31, 34, 56, 67 pgs 119-120: 8, 14, 23, 25, 29
		AFTER LESSON 7		pgs 87-89: 22, 29, 34 pgs 109-111: 16, 21, 26, 29, 30

8	**M-8** **30 Aug**	**GRADED REVIEW I** [Room: _______]	*Arrive NLT 0645 hrs* *Do not come to class 30, 31 Aug*

BLOCK II - Functions, Graphs and Transformations

Lesson	Date	Topic	ME Due	Other Assignments Due
9	1, 5 Sep	Variation	2.9	None
10	6, 7 Sep	Graphs of Equations	3.1	pgs 124-125: 9, 12, 15, 17, 18, 22, 25, 31
11	8, 11 Sep	Mathematica Tour	None	pgs 137-138: 13, 15, 36, 43, 44, 45, 50, 57, 58
12	12, 13 Sep	Functions	3.3	None
13	14, 15 Sep	GROUP SOLVE	None	pgs 151-152: 3, 9, 10, 11, 15, 19, 20, 23, 28, 29
14	18, 19 Sep	Linear Functions	3.4	None
15	20, 21 Sep	Graphs & Applications of Functions	3.5	pgs 164-166: 1, 5, 8, 19, 22 29,30, 53, 54, 57
16	22, 25 Sep	Combinations of Functions	3.7	pgs 175-178: 28, 31, 41, 43, 45,46, 48, 60, 67, 68
17	26, 27 Sep	Transformations of Functions	3.8	pgs 196-197: 3, 5, 11, 13, 18 25, 28, 31, 35, 37
18	28, 29 Sep	Review	None	pgs 204-205: 21, 27, 35, 37, 41, 42, 43, 44

19	T-19 **3 Oct**	**GRADED REVIEW II** [Room: _______]	*Arrive NLT 0645 hrs* *Do not come to class 2, 3 Oct*

BLOCK III - Polynomial and Rational Functions

Lesson	Date	Topic	ME Due	Other Assignments Due
20	4, 5 Oct	Quadratic Functions	4.1	None
21	6, 10 Oct	Sets, Sequences, & Inequalities	4.2	pgs 222-223: 7, 11, 18, 23, 35, 38, 43, 47, 52
22	11, 12 Oct	Equations & Inequal with absolute value	4.3	pg 229: 5, 6, 11, 15, 21, 24, 29, 32, 40, 43
23	13, 16 Oct	Polynomial & Rational Inequalities	4.4	pg 234: 1, 3, 6, 15, 19, 21, 29, 32, 36, 44

24	17, 18 Oct	Graphs of Polynomial Functions	4.5	pgs 239-240: 1, 8, 13, 21, 22, 29, 32, 37, 49
25	19, 20 Oct	GROUP SOLVE 2	None	pgs 246-247: 5, 6, 9, 13, 16, 25, 27, 28, 36
26	23, 24 Oct	Division of Polys Theorem - Roots	4.6 4.7	Written Exercise 2
27	25, 26 Oct	Rational Functions	4.9	pgs 252-253: 3, 35, 40 pgs 261-262: 5, 14, 25, 29, 33, 36, 40
28	27, 30 Oct	Review	None	pgs 278-279: 7, 13, 16, 19, 21, 27, 33, 34

29 Oct	**M-29** [Room: _______]	**GRADED REVIEW III** *Arrive NLT 0645 hrs* *Do not come to class 31 Oct, 1 Nov*	**31**

BLOCK IV - Trigonometric Functions and Identites

Lesson	Date	Topic	ME Due	Other Assignments Due
30	2, 3 Nov	Unit Circle	6.1	None
31	6, 7 Nov	Sine and Cosine Functions	6.2	pgs 358-359: 1, 3, 6, 9, 10, 13, 17, 18
32	8, 9 Nov	Other Basic Circular Functions	6.3	pgs 369-370: 9, 12, 38, 45, 48, 54, 63, 64, 65, 73
33	13, 14 Nov	Angles and Rotations	6.4	pgs 380-381: 3, 6, 11, 14, 20, 22, 24, 25, 27, 30
34	15, 16 Nov	Trig Functions Involving Angles or Rotations	6.5 (odd)	Course Project Due pgs 393-395: 47, 48, 57, 60, 75, 86, 91, 96
35	17, 20 Nov	Trig Transformations	6.7	pgs 407-408: 18, 20, 23, 30, 35, 37, 38, 39, 41
36	21, 22 Nov	Trig Manipulations and Identities	7.1	pgs 432-433: 1, 7, 12, 21, 25, 28, 39, 40
37	27, 28 Nov	Sum and Difference Identities Some Important Identities	7.2 7.4	pgs 448-450: 5, 9, 14, 21, 31, 32, 37, 45, 52

38	29, 30 Nov	Proving Trig Identities	7.5	pgs 455-457: 13, 16, 22, 25 pgs 468-469: 3, 6, 15, 29, 34, 37
39	1, 4 Dec	Group Solve 3	None	pgs 476-478: 3, 5, 9, 12, 20, 29, 34, 37
40	5, 6 Dec	Review	None	None
41	**M-41** **7 Dec**	[Room: _______]	**GRADED REVIEW IV**	*Arrive NLT 0645 hrs* *Do not come to class 7, 8 Dec*
42	11, 12 Dec	Wrap-Up/Review	None	None

Course Syllabus
Treatment Group

TEXT: Algebra and Trigonometry, by Keedy, Bittinger, & Beecher, Sixth Edition, 1993

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2	14, 15 Aug	Factoring Rational Expressions	1.5 (even) 1.6	pg 18: 23, 26, 30
3	16, 17 Aug	Radical Notation Rational Exponents	1.7 (even) 1.8 (odd)	pg 18: 43 pg 32: 9, 11, 13
4	18, 21 Aug	Solving Equations & Inequalities	2.1	pg 32: 18 pg 40: 15 pg 49: 7, 26, 41, 64
5	22, 23 Aug	Rational Equations	2.2	pg 19: 99 pg 50: 75 pg 55: 30 pg 70: 7, 11, 14, 16
6	24, 25 Aug	Quadratic Equations Equations Reducible to Quadratic	2.5 2.8	pg 40: 22 pg 70: 17, 24 pg 76: 8, 15, 18
7	28, 29 Aug	Formulas for Problem Solving	2.3	pg 55: 36 pg 76: 19 pg 105: 23, 31, 34, 56
		AFTER LESSON 7		pg 19: 104 pg 71: 45 pg 88: 22, 29, 34 pg 105: 67 pg 119: 8

8	**M-8** **30 Aug**	**GRADED REVIEW I** [Room: _______]	*Arrive NLT 0645 hrs* *Do not come to class 30, 31 Aug*

Block II - Functions, Graphs and Transformations

Lesson	Date	Topic	ME Due	Other Assignments Due
9	1, 5 Sep	Variation	2.9	None
10	6, 7 Sep	Graphs of Equations	3.1	pg 41: 34 pg 76: 22 pg 109: 16 pg 124: 9, 12, 15
11	8, 11 Sep	Mathematica Tour	None	pg 55: 37 pg 119: 14 pg 125: 17 pg 137: 13, 15, 36
12	12, 13 Sep	Functions	3.3	None
13	14, 15 Sep	GROUP SOLVE 1	None	pg 71: 47 pg 110: 21 pg 137: 43, 44 pgs 151-152: 3, 9, 10, 11
14	18, 19 Sep	Linear Functions	3.4	None
15	20, 21 Sep	Graphs & Applications of Functions	3.5	pg 76: 25 pg 125: 18 pg 152: 15, 19 pg 164: 1, 5, 8, 19
16	22, 25 Sep	Combinations of Functions	3.7	pg 19: 115 pg 119: 23 pg 138: 45 pg 164: 22, 29 pgs 176-177: 28, 31, 41, 43
17	26, 27 Sep	Transformations of Functions	3.8	pg 41: 41 pg 110: 26 pg 152: 20 pg 177: 45, 46 pgs 196-197: 3, 5, 11, 13, 37
18	28, 29 Sep	Review	None	pg 56: 54 pg 125: 22 pg 164: 30 pg 196: 18, 25 pgs 204-205: 21, 27, 35, 44

19	T-19 **3 Oct**	**GRADED REVIEW II** [Room: _______]	*Arrive NLT 0645 hrs* *Do not come to class 2, 3 Oct*

BLOCK III - Polynomial and Rational Functions

Lesson	Date	Topic	ME Due	Other Assignments Due
20	4, 5 Oct	Quadratic Functions	4.1	None
21	6, 10 Oct	Sets, Sequences, & Inequalities	4.2	pg 71: 50 pg 138: 50 pgs 177-178: 48, 68 pg 204: 37 pg 222: 7, 11, 18
22	11, 12 Oct	Equations & Inequal with absolute value	4.3	pg 76: 27 pg 152: 23 pg 196: 28 pg 222: 23, 35, 52 pg 229: 5, 6, 11, 15
23	13, 16 Oct	Polynomial & Rational Inequalities	4.4	pg 119: 25 pg 165: 53 pg 204: 41 pg 229: 21, 24, 43 pg 234: 1, 3, 6, 15
24	17, 18 Oct	Graphs of Polynomial Functions	4.5	pg 110: 29 pg 177: 60 pg 222: 38 pg 234: 19, 21, 44 pg 239: 1, 8, 13
25	19, 20 Oct	GROUP SOLVE 2	None	pg 125: 25 pg 196: 31 pg 229: 29 pg 239: 21, 22, 49 pg 246: 5, 6, 9, 36
26	23, 24 Oct	Division of Polys Theorem - Roots	4.6 4.7	Written Exercise 2
27	25, 26 Oct	Rational Functions	4.9	pg 138: 57 pg 204: 42 pg 234: 29 pg 246: 13, 16 pgs 252-253: 3, 35, 40 pg 261: 5, 36, 40

28	27, 30 Oct	Review	None	pg 19: 116 pg 152: 28 pg 222: 43 pg 239: 29 pg 261: 14, 25 pg 278: 7, 13, 16, 34

29	**M-29** **31 Oct**	**GRADED REVIEW III** *Arrive NLT 0645 hrs* [Room: _______]*Do not come to class 31 Oct, 1 Nov*

BLOCK IV - Trigonometric Functions and Identites

Lesson	Date	Topic	ME Due	Other Assignments Due
30	2, 3 Nov	Unit Circle	6.1	None
31	6, 7 Nov	Sine and Cosine Functions	6.2	pg 41: 47 pg 165: 54 pg 229: 32 pg 246: 25 pg 278: 19, 33 pgs 358-359: 1, 3, 6, 17, 18
32	8, 9 Nov	Other Basic Circular Functions	6.3	pg 56: 57 pg 165: 57 pg 178: 67 pg 234: 32 pg 261: 29 pg 359: 9 pgs 369-370: 9, 12, 38, 45, 65, 73
33	13, 14 Nov	Angles and Rotations	6.4	pg 71: 54 pg 197: 35 pg 239: 32 pg 278: 21 pgs 369-370: 48, 54 pgs 380-381: 3, 6, 11, 14, 27, 30
34	15, 16 Nov	Trig Functions Involving Angles or Rotations	6.5 (odd)	Course Project Due pg 76: 34 pg 205: 43 pg 246: 27 pgs 358-359: 10 pg 380: 20, 22 pgs 393-394: 47, 48, 57, 86, 91

35	17, 20 Nov	Trig Transformations	6.7	pg 119: 29 pg 222: 47 pg 261: 33 pgs 369-370: 63 pgs 393-394: 60, 96 pgs 407-408: 18, 20, 23, 38, 41
36	21, 22 Nov	Trig Manipulations and Identities	7.1	pg 110: 30 pg 229: 40 pg 278: 27 pg 381: 24 pgs 407-408: 30, 35, 39 pg 432: 1, 7, 12, 40
37	27, 28 Nov	Sum and Difference Identities Some Important Identities	7.2 7.4	pg 125: 31 pg 234: 36 pgs 358-359: 13 pg 393: 75 pg 432: 21, 28, 39 pgs 448-450: 5, 9, 14, 32, 45, 52
38	29, 30 Nov	Proving Trig Identities	7.5	pg 138: 58 pg 239: 37 pgs 369-370: 64 pg 408: 37 pgs 448-449: 21, 31, 37 pg 456: 13, 16, 22, 25 pgs 469-469: 15, 34, 37
39	1, 4 Dec	Group Solve 3	None	pg 152: 29 pg 246: 28 pg 381: 25 pg 432: 25 pgs 468-469: 3, 6, 29 pg 476: 3, 5, 9, 12, 20, 29, 34, 37
40	5, 6 Dec	Review	None	None
41	**M-41** **7 Dec**	**GRADED REVIEW IV** [Room: _______]	*Arrive NLT 0645 hrs* *Do not come to class 7, 8 Dec*	
42	11, 12 Dec	Wrap-Up/Review	None	None

APPENDIX C

Observation Checksheet

Observer: ________________ Instructor: ________________
Section: ________ Period: ________ Date: ___________

1. Homework Review
Did the instructor go over homework problems?

Homework was reviewed from ___:___ to ___:___; total minutes: _____

Which problems?

What was the instructor's method for reviewing homework?

2. Homework Collection
Did the instructor collect homework?

Did the students seem to expect that homework would be collected?

3. Homework Distribution
Did the instructor distribute homework that had been checked for correctness and completion?

When did this distribution occur?

Did the students ask questions about the scores? (if yes, list questions)

Did the instructor discuss the scoring rubric?

4. Lesson
Were new lesson topics covered in accordance with the syllabus? (if no, explain)

5. Assigning Homework
Was new homework assigned according to the syllabus? (if no, explain)

Student Mid-Semester Survey

Directions: Your answers to these questions will be used to improve our teaching. Use the codes below to answer the questions. Please answer the questions honestly. Your instructor will not have access to individual surveys.

Use these codes to answer questions 1 through 7:

1 = strongly disagree 2 = disagree 3 = neutral 4 = agree 5 = strongly agree

1. Homework problems for this course are carefully selected.

2. Daily homework problems are related only to topics covered in the most current lesson.

3. Homework should be collected to give students feedback but NOT graded.

4. Homework helps me prepare for the hourly exams.

5. The homework schedule in the syllabus is easy to follow.

6. Homework should be collected for a grade.

7. Daily homework consists of problems related to the current lesson mixed with problems related to topics covered in previous lessons.

Use these codes to answer questions 8 through 12:

1 = never 2 = rarely 3 = sometimes 4 = frequently 5 = always

8. Homework is collected and check for correctness and completion.

9. Homework is returned promptly.

10. My instructor spends class time going over homework problems.

11. I work with others on the homework.

12. I work on homework with students who use a different homework syllabus.

Student End of Semester Survey

Directions: Your answers to these questions will be used to improve our teaching. Use the codes below to answer the questions. Please answer the questions honestly. Your instructor will not have access to individual surveys.

Use these codes to answer questions 1 through 6:
1 = strongly disagree 2 = disagree 3 = neutral 4 = agree 5 = strongly agree

1. Homework problems for this course were carefully selected.

2. Daily homework problems were related only to topics covered in the most current lesson.

3. Homework helped me prepare for the hourly exams.

4. The homework schedule in the syllabus was easy to follow.

5. Homework should be collected for a grade.

6. Daily homework consisted of problems related to the current lesson mixed with problems related to topics covered in previous lessons.

Use these codes to answer questions 7 through 11:

1 = never 2 = rarely 3 = sometimes 4 = frequently 5 = always

7. Homework was collected and checked for correctness and completion.

8. Homework was returned promptly.

9. My instructor spent class time going over homework problems.

10. I worked with others on the homework.

11. I worked on homework with students who used a different homework syllabus.

12. The grade I expect to receive in this course is:
1 = A 2 = B 3 = C 4 = D 5 = F

Instructor Mid-Semester Survey - Part I

Name: ________________________ Section(s): ______________

Your answers to these questions will be used to verify the experimental and control treatments and to support the results of the experiment with anecdotal data. Please answer all questions honestly. Thanks for your help.

Please circle your responses.

1. I assign homework according to the syllabus.

   strongly disagree disagree neutral agree strongly agree

2. I spend class time going over homework problems.

   strongly disagree disagree neutral agree strongly agree

3. Going over homework is a normal part of my lesson plan.

   strongly disagree disagree neutral agree strongly agree

4. The homework scores provided by the researcher are useful to me.

   strongly disagree disagree neutral agree strongly agree

5. The homework scores are useful to my students.

   strongly disagree disagree neutral agree strongly agree

6. My pre-lesson workload was increased as a result of this experiment.

   strongly disagree disagree neutral agree strongly agree

7. My post-lesson workload (workload resulting from tests, grading, and data collection) was increased as a result of this experiment.

   strongly disagree disagree neutral agree strongly agree

8. When questioned by students, I freely discuss this experiment in class.

   strongly disagree disagree neutral agree strongly agree

Question 9 is for distributed practice treatment instructors only.

9. My planning was complicated by the use of the distributed practice homework model.

   strongly disagree disagree neutral agree strongly agree

Instructor Mid-Semester Survey - Part II

Name: ______________________________

1. On average, how much time do you spend each lesson reviewing homework?

2. How do you go over the homework?

3. How do you use the homework completion scores?

4. What is your policy for accepting "late" homework?

5. What is your policy for excusing students from homework (i.e. not giving a zero)?

<u>The following questions are based on your answers to Part I survey items. Please answer all questions that are circled.</u>

6. How has your pre-lesson workload increased as a result of this experiment?

7. How has your post-lesson workload increased as a result of this experiment?

8. What have you told students about the experiment?

9. How has the distributed practice model complicated your planning?

Instructor End of Semester Survey - Part I

Name: ______________________ Section(s): ____________

Your answers to these questions will be used to verify the experimental and control treatments and to support the results of the experiment with anecdotal data. Please answer all questions honestly. Thanks for your help.

Please circle your responses.

1. I assigned homework according to the syllabus.

   strongly disagree disagree neutral agree strongly agree

2. I spent class time going over homework problems.

   strongly disagree disagree neutral agree strongly agree

3. Going over homework was a normal part of my lesson plan.

   strongly disagree disagree neutral agree strongly agree

4. The homework scores provided by the researcher were useful to me.

   strongly disagree disagree neutral agree strongly agree

5. This experiment increased my knowledge of educational research.

   strongly disagree disagree neutral agree strongly agree

6. This experiment increased my knowledge of learning theory.

   strongly disagree disagree neutral agree strongly agree

7. My pre-lesson workload was increased as a result of this experiment.

   strongly disagree disagree neutral agree strongly agree

8. My post-lesson workload (workload resulting from tests, grading, and data collection) was increased as a result of this experiment.

   strongly disagree disagree neutral agree strongly agree

9. I expect the results of this experiment to be directly applicable to my teaching.

   strongly disagree disagree neutral agree strongly agree

10. When questioned by students, I freely discussed this experiment in class.

strongly disagree disagree neutral agree strongly agree

<u>Question 11 is for distributed practice treatment instructors only.</u>

11. My planning was complicated by the use of the distributed practice homework model.

strongly disagree disagree neutral agree strongly agree

Instructor End of Semester Survey - Part II

Name: ______________________________

Answer questions 1, 2, and 3 ONLY if your answers have changed since the mid-semester survey.

1. On average, how much time do you spend each lesson reviewing homework?

2. How do you go over the homework?

3. How do you use the homework completion scores?

The following questions are based on your answers to Part I survey items. Please answer all questions that are circled.

4. How has your pre-lesson workload increased as a result of this experiment?

5. How has your post-lesson workload increased as a result of this experiment?

6. What have you told students about the experiment?

7. How has the distributed practice model complicated your planning?

8. How will the results of this experiment affect your teaching?

9. How has this experiment increased your knowledge of educational research?

10. How has this experiment increased your knowledge of learning theory?

APPENDIX D

Date: Mon, 17 Apr 1995 17:29:33 -0400 (EDT)
From: HIRSCH@wmich.edu
To: mrevak@winnie.fit.edu
Subject: Dissertation Study

Dear Ms. Revak:

You have my permission to use a modification of the distributive practice table in our IJMEST article for your dissertation study.

We have not done any recent work in this area.

Best wishes on your study.

Regards,

Chris Hirsch

Date: Mon, 10 Apr 1995 16:17:08 -0400 (EDT)
From: "John C. Peterson" <JPETERSON@STCC.CC.TN.US>
To: mrevak@winnie.fit.edu
Subject: Homework assignment model request

I have no objection to your using the homework assignment models to describe your experimental and control groups. I am interested in hearing more about your study. Would it be possible for you to send me a synopsis?

It has been awhile since I have done any work in this area. I will send you some other references (which you probably already have) in a few days.

Best of luck. Let me know what you find out.

John C. Peterson